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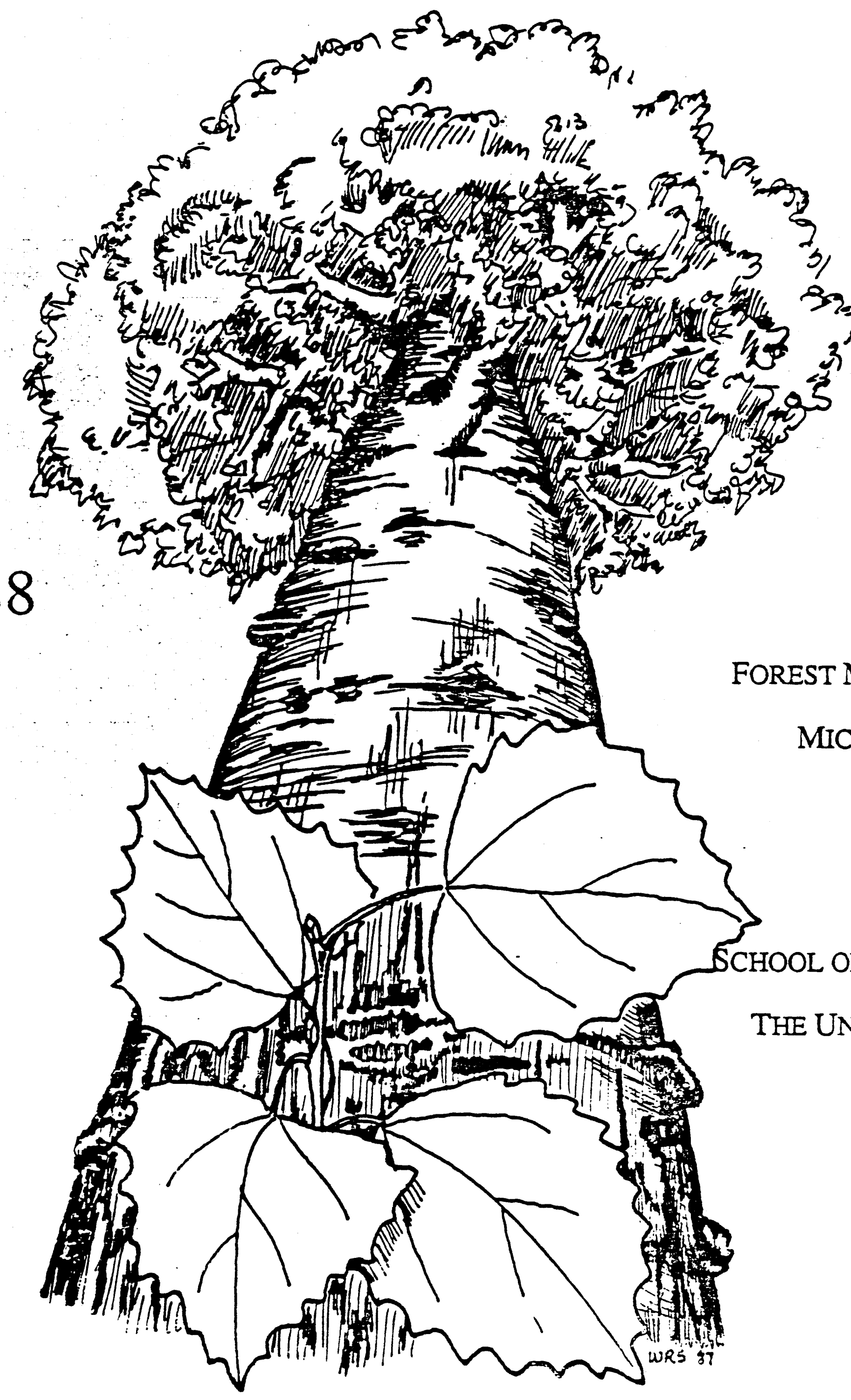

1988

FOREST MANAGEMENT DIVISION
MICHIGAN DEPARTMENT OF
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Management Summary

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot sawtimber volume-basal area ratio (VBAR in cu.ft./sq.ft. or bd.ft./sq.ft.) equations were developed for aspen in Michigan. Data used to develop these equations were collected from 24 aspen stands in Michigan (12 stands each from the Upper and Lower Peninsulas). Four stands each were sampled from each of the 6 state forests in Michigan.

VBAR multiple linear regression equations using diameter at breast height (DBH) and merchantable height yielded somewhat higher coefficients of determination (R^2), lower standard errors of estimate ($s_{y \cdot x}$), and approximately the same accuracy compared to VBAR equations using height independent variables. However, the differences between the 2 sets of equations are relatively small, indicating that the use of the simpler height VBAR equations is justified for most cruising situations. The new Scribner and International 1/4-inch board-foot VBAR equations yield, in general, VBAR values larger than respective tabled values in Edminster et al. (1982). However, the merchantability specifications of the two sets of equations are different. The new International 1/4-inch board-foot VBAR equation yields VBAR values relatively close to the values presently used by the DNR. In general, estimates using the new VBAR equation would be no more than 3% less than those based on the current DNR VBARs. Both sets of values are based on similar merchantability limits.

We recommend the use of the following VBAR equations in most cruising situations for aspen:

1. Cubic-foot VBAR

$$\hat{VBAR}_C = 7.900 + 4.129 \cdot H - 4.453 \cdot \frac{1}{H}$$

2. Doyle board-foot VBAR

   $\hat{VBAR}_D = -13.110+21.966 \cdot H+8.952 \cdot \frac{1}{H}$

3. International 1/4-inch board-foot VBAR

   $\hat{VBAR}_I = 21.502+27.507 \cdot H-12.984 \cdot \frac{1}{H}$

4. Scribner board-foot VBAR

   $\hat{VBAR}_S = 15.186+27.204 \cdot H-9.187 \cdot \frac{1}{H}$

In the above equations, H is the merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit.

Multiple linear regression equations were also developed to predict one type of VBAR from another for the 4 types of VBARs examined in this study.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - VOLUME-BASAL AREA RATIO EQUATIONS

DATE - 21 Sept. 88

TITLE - Sawtimber Volume-Basal Area Ratio Equations for Aspen in Michigan

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Background

The Michigan Department of Natural Resources (DNR) developed sawtimber volume basal area ratios (VBARs) in bd.ft. per sq.ft. to be used in estimating International 1/4-inch board-foot volume using prism cruising. DNR Tally Sheet R 4145 was developed from these VBARs. Sawtimber cubic-foot and Scribner and International 1/4-inch board-foot VBARs have been developed for aspen in Colorado (Edminster et al. 1982).

Purpose

The purpose of this paper is to present new sawtimber cubic-foot and Doyle, International 1/4-inch, and Scribner VBAR equations and tables for aspen in Michigan.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 1381 trees from 24 stands as follows:

1) 677 trees from 12 stands in the Upper Peninsula (i.e., 4 stands each in the Copper, Escanaba River, and Lake Superior state forests), and
2) 704 trees from 12 stands in the Lower Peninsula (i.e., 4 stands each in the Mackinaw, Au Sable, and Marquette state forests).

Measurements were taken on 728 bigtooth aspen and 653 trembling aspen trees. Stands were selected from the 6 state forests to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1986.

For the 24 stands, site index varied from 51 to 79, age varied from 47 to 70 years, basal area/acre varied from 70 to 186 sq.ft., average DBH varied from 7.7 to 11.9 in., average total height varied from 52.2 to 77.5 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 3.4 to 7.7 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b) developed pulpwood, sawtimber, and residual pulpwood cubic-foot VBAR equations from the total data set described above.

Sawtimber trees were defined as trees that had at least one 100-in. stick with a minimum inside bark top diameter no smaller than 7.6 inches. Sawtimber mechantable height is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches. There was a total of 946 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into 2 pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. For each 100-in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V=\frac{(B+b)L}{2}$ (Avery and Burkhart 1983)

Doyle: $V=0.5D^2-4.0D+8.0$ (Husch et al. 1982)

International 1/4-inch: $V=0.905(0.44D^2-1.20D-0.30)$ (Husch et al. 1982)

Scribner: $V=0.395D^2-0.99D-2.15$ (Bruce and Schumacher 1950)

where

V = volume in cubic feet or board feet,

L = length of stick (100 in.) in ft.,

B = cross-sectional area inside bark of large end of the stick in sq.ft.,

b = cross-sectional area inside bark of small end of the stick in sq.ft., and

D = diameter of small end of stick inside bark in inches.

See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the 3 board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 7.6-in. top diameter limit. Sawtimber VBARs were obtained for each tree by dividing cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot volumes of the tree by the basal area in sq.ft. of the tree at 4.5 ft. from the ground.

VBAR was regressed on various forms of sawtimber merchantable height and DBH using multiple linear regression.

Results

The data set used to develop the regression equations consisted of 750 trees (≅80%) selected at random from the total of 946 trees. Fowler and Hussain (1987b) found no significant differences between pulpwood cubic-foot volume equations of the 6 forest areas and 2 species, so the data for both species and all stands were pooled before developing VBAR regression equations. All equations were based on the 750 trees with an average DBH=10.8 in. (range: 8.0 to 16.7), average merchantable height=2.8 sticks (range: 1 to 7), average cubic-foot VBAR=17.34 cu.ft./sq.ft. (range: 6.53 to 37.48), average Doyle board-foot VBAR=52.87 bd.ft./sq.ft. (range: 13.16 to 174.49), average International 1/4-inch VBAR=92.43 bd.ft./sq.ft. (range: 29.41 to 229.03), and average Scribner board-foot VBAR=87.10 bd.ft./sq.ft. (range: 26.70 to 223.46).

Cubic-foot and board-foot VBAR prediction equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated that the following prediction equations compared favorably to all other equations examined for cubic-foot and the 3 board-foot VBARs:

1. Height independent variables

   $$\widehat{VBAR} = \hat{\beta}_0 + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H}$$

2. Height and DBH independent variables

   $$\widehat{VBAR} = \hat{\beta}_0 + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H} + \hat{\beta}_3 D + \hat{\beta}_4 \frac{1}{D},$$

where $\widehat{VBAR}$ is predicted VBAR, H is merchantable height in 100-in. sticks to a 7.6-in. top diameter limit, and DBH is in inches. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ are the sample regression coefficients related to the independent variables.

Table 1 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs along with standard errors of the estimate ($s_{y \cdot x}$), and coefficients of determination (R^2) for the height only models. Table 2 shows the cubic-foot and 3 board-foot VBARs for various merchantable heights based on Equations 1-4. Table 2 also shows International 1/4-inch VBARs presently used by the DNR (Tally Sheet R 4145). The new International 1/4-inch VBARs are 14.3, 2.8, 2.0, 1.5, and 0.0% smaller than the DNR VBARs for trees with 1, 2, 3, 4, and 5 sticks, respectively. For 6-stick trees, the new VBAR is 0.55% larger than the DNR VBAR. In general, the new VBARs are in relatively close agreement with the DNR VBARs.

Table 3 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs for the height and diameter models. Note that R^2 and $s_{y \cdot x}$ for these equations are somewhat larger and smaller, respectively, than for the respective equations based on height only (Table 1). A sawtimber cubic-foot VBAR table based on Equation 5 is shown in Table 4, and Doyle, International 1/4-inch, and Scribner board-foot VBAR tables are shown in Tables 5, 6, and 7 based on Equations 6, 7, and 8, respectively.

The values in Table 6 (International 1/4-inch board-foot VBARs) are, in general, larger than the values in Table 11 of Edminster et al. (1982) except for small merchantable heights. The merchantable height where the values in our Table 6 become larger than those in Table 11 of Edminster et al. increases as DBH increases. The values in Table 7 (Scribner board-foot VBARs) are larger than the values in Table 7 of Edminster et al.

In comparing the 2 sets of tables, it should be kept in mind that our tables assume a 0.5-ft. stump, an approximate minimum top diameter of 7.6 in., and log lengths of 8 ft., whereas Edminster et al. assume a 1.0-ft. stump, an approximate 6.0 in. top diameter, and log lengths of 16 and 8 ft.

Table 1. Estimated intercepts ($\hat{\beta}_0$), regression coefficents $\hat{\beta}_1$ and $\hat{\beta}_2$, standard errors of the estimate ($s_{y \cdot x}$), and coefficients of multiple determination (R^2) for the cubic-foot and three board-foot VBAR prediction equations with independent variables based on height only.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	$s_{y \cdot x}$
(1) Cubic-foot[a]	7.900	4.129	- 4.453	0.977	1.06
(2) Doyle[b]	-13.110	21.966	8.952	0.965	5.40
(3) International 1/4-inch[c]	21.502	27.507	-12.984	0.973	7.67
(4) Scribner[d]	15.186	27.204	- 9.187	0.974	6.65

[a] $\hat{VBAR} = 7.900+4.129 \cdot H-4.453 \cdot \frac{1}{H}$

[b] $\hat{VBAR} = -13.110+21.966 \cdot H+8.952 \cdot \frac{1}{H}$

[c] $\hat{VBAR} = 21.502+27.507 \cdot H-12.984 \cdot \frac{1}{H}$

[d] $\hat{VBAR} = 15.186+27.204 \cdot H-9.187 \cdot \frac{1}{H}$

Table 2. VBARs in cu.ft./sq.ft. or bd.ft./sq.ft. for the four types of volume for various values of merchantable height. International 1/4-inch VBARs are also given from DNR Tally Sheet R 4145.

Height (sticks)	Cu.ft./Sq.ft.	Bd.ft./Sq.ft.			
		Doyle	Scribner	International 1/4-inch	
				New	DNR
1	7.58	18	33	36	42
2	13.93	35	65	70	72
3	18.80	56	94	100	102
4	23.30	77	122	128	130
5	27.65	99	149	156	156
6	31.93	120	177	184	183
7	36.17	142	204	212	---
8	40.38	164	232	240	---
9	44.57	186	259	268	---
10	48.74	207	286	295	---

Table 3. Estimated intercepts ($\hat{\beta}_0$), regression coefficents $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$, standard errors of the estimate ($s_{y\cdot x}$), and coefficients of multiple determination (R^2) for the cubic-foot and three board-foot VBAR prediction equations with independent variables based on height and diameter.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2	$s_{y\cdot x}$
(5) Cubic-foot[a]	32.613	5.150	-3.048	-1.700	-105.254	0.987	0.82
(6) Doyle[b]	-84.948	20.278	4.981	4.073	366.354	0.967	5.30
(7) International 1/4-inch[c]	94.387	32.146	-8.697	-6.117	-233.663	0.979	6.81
(8) Scribner[d]	71.418	30.716	-5.886	-4.673	-183.466	0.977	6.19

[a] $\widehat{VBAR} = 32.613+5.150\cdot H-3.048\cdot\frac{1}{H} -1.700\cdot D-105.254\cdot\frac{1}{D}$

[b] $\widehat{VBAR} = -84.948+20.278\cdot H+4.981\cdot\frac{1}{H} +4.073\cdot D+366.354\cdot\frac{1}{D}$

[c] $\widehat{VBAR} = 94.387+32.146\cdot H-8.697\cdot\frac{1}{H} -6.117\cdot D-233.663\cdot\frac{1}{D}$

[d] $\widehat{VBAR} = 71.418+30.716\cdot H-5.886\cdot\frac{1}{H} -4.673\cdot D-183.466\cdot\frac{1}{D}$

Table 4. VBAR table showing cu.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 5).

DBH (inches)	Merchantable Height in Sticks 1	2	3	4	5	6	7	8	9
9	7.72	14.39	20.05	25.46	30.76				
10	7.19	13.86	19.52	24.93	30.23	35.48			
11	6.45	13.12	18.78	24.18	29.48	34.74			
12	5.54	12.22	17.88	23.28	28.58	33.83	39.06		
13	4.52	11.19	16.85	22.25	27.56	32.81	38.03	43.24	
14	3.40	10.07	15.73	21.13	26.44	31.69	36.91	42.11	
15	2.20	8.87	14.53	19.93	25.24	30.49	35.71	40.92	46.11
16		7.61	13.27	18.67	23.98	29.23	34.45	39.65	44.85
17		6.30	11.96	17.36	22.66	27.91	33.14	38.34	43.53
18		4.94	10.60	16.00	21.31	26.56	31.78	36.98	42.18
19		3.55	9.21	14.61	19.91	25.17	30.39	35.59	40.78
20		2.12	7.78	13.19	18.49	23.74	28.96	34.17	39.36
21			6.33	11.74	17.04	22.29	27.52	32.72	37.91
22			4.86	10.27	15.57	20.82	26.04	31.25	36.44
23			3.37	8.77	14.08	19.33	24.55	29.76	34.95
24			1.86	7.27	12.57	17.82	23.04	28.25	33.44
25			0.34	5.74	11.04	16.29	21.52	26.72	31.91

Table 5. VBAR table showing Doyle bd.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 6).

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
9	17.7	35.5	54.9	74.8	94.8				
10	17.7	35.5	54.9	74.8	94.8	114.9			
11	18.4	36.2	55.7	75.5	95.5	115.7			
12	19.7	37.5	57.0	76.8	96.8	117.0	137.1		
13	21.4	39.2	58.7	78.5	98.6	118.7	138.8	159.0	
14	23.5	41.3	60.7	80.6	100.6	120.7	140.9	161.1	
15	25.8	43.6	63.1	83.0	103.0	123.1	143.2	163.4	183.6
16		46.2	65.6	85.5	105.5	125.6	145.8	166.0	186.2
17		48.9	68.3	88.2	108.2	128.3	148.5	168.7	188.9
18		51.8	71.2	91.1	111.1	131.2	151.4	171.6	191.8
19		54.8	74.2	94.1	114.1	134.2	154.4	174.6	194.8
20		57.9	77.3	97.2	117.2	137.3	157.5	177.7	197.9
21			80.5	100.4	120.4	140.5	160.7	180.9	201.1
22			83.8	103.7	123.7	143.8	164.0	184.2	204.4
23			87.2	107.0	127.0	147.2	167.3	187.5	207.7
24			90.6	110.4	130.5	150.6	170.7	190.9	211.1
25			94.0	113.9	133.9	154.0	174.2	194.4	214.6

Table 6. VBAR table showing International 1/4-inch bd.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 7).

DBH (inches)	Merchantable Height in Sticks 1	2	3	4	5	6	7	8	9
9	36.8	73.3	106.9	139.8	172.4				
10	33.3	69.8	103.4	136.3	168.8	201.3			
11	29.3	65.8	99.4	132.3	164.8	197.3			
12	25.0	61.5	95.1	127.9	160.5	192.9	225.3		
13	20.3	56.8	90.4	123.3	155.9	188.3	220.7	253.0	
14	15.5	52.0	85.6	118.5	151.0	183.5	215.8	248.1	
15	10.5	47.0	80.6	113.5	146.0	178.5	210.8	243.1	275.4
16		41.9	75.5	108.3	140.9	173.3	205.7	238.0	270.3
17		36.6	70.2	103.1	135.6	168.1	200.4	232.7	265.0
18		31.2	64.8	97.7	130.3	162.7	195.1	227.4	259.6
19		25.8	59.4	92.3	124.9	157.3	189.6	221.9	254.2
20		20.3	53.9	86.8	119.4	151.8	184.1	216.4	248.7
21			48.3	81.2	113.8	146.2	178.6	210.9	243.1
22			42.7	75.6	108.2	140.6	173.0	205.3	237.5
23			37.1	69.9	102.5	135.0	167.3	199.6	231.9
24			31.4	64.3	96.8	129.3	161.6	193.9	226.2
25			25.7	58.5	91.1	123.5	155.9	188.2	220.5

Table 7. VBAR table showing Scribner bd.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 8).

DBH (inches)	Merchantable Height in Sticks 1	2	3	4	5	6	7	8	9
9	33.8	67.5	99.2	130.4	161.4				
10	31.2	64.8	96.5	127.7	158.7	189.7			
11	28.2	61.8	93.5	124.7	155.7	186.7			
12	24.9	58.5	90.2	121.4	152.5	183.4	214.2		
13	21.4	55.0	86.7	117.9	149.0	179.9	210.7	241.5	
14	17.7	51.4	83.1	114.3	145.3	176.2	207.1	237.9	
15	13.9	47.6	79.3	110.5	141.5	172.4	203.3	234.1	264.9
16		43.7	75.4	106.6	137.6	168.5	199.4	230.2	261.0
17		39.7	71.4	102.6	133.6	164.5	195.4	226.2	257.0
18		35.6	67.3	98.5	129.5	160.4	191.3	222.1	252.9
19		31.5	63.2	94.4	125.4	156.3	187.1	218.0	248.8
20		27.3	59.0	90.2	121.2	152.1	183.0	213.8	244.6
21			54.7	85.9	117.0	147.9	178.7	209.5	240.3
22			50.5	81.7	112.7	143.6	174.4	205.3	236.1
23			46.1	77.4	108.4	139.3	170.1	201.0	231.8
24			41.8	73.0	104.0	134.9	165.8	196.6	227.4
25			37.4	68.6	99.7	130.6	161.4	192.2	223.0

Predicting one type of VBAR from another

Multiple linear regression equations were developed to predict one type of VBAR from another using the 750 trees in the prediction data set. Equations were developed for predicting cubic-foot VBAR (CR) as a function of Doyle (DR), International 1/4-inch (IR), and Scribner (SR) VBARs, DR as a function of CR, IR, and SR, IR as a function of CR, DR, and SR, and SR as a function of CR, DR, and IR. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 8.

Validation

The data set used to validate the prediction equations consisted of 196 trees, the remaining approximately 20% of the total of 946 trees. For each VBAR equation, the average relative error as a percent ($\overline{RE}$) was calculated where

$$\overline{RE} = \sum_{i=1}^{n} RE_i \Big/ n$$

and $RE_i = [(\hat{VBAR}_i - VBAR_i)/VBAR_i] \times 100$, $\hat{VBAR}_i$ and $VBAR_i$ are the predicted and actual VBARs for the i^{th} tree, and n is the number of trees in the validation data set (n=196). The relative error as a percent for the sum of the predicted VBARs was also calculated where

$$RE_S = [(\sum_{i=1}^{n} \hat{VBAR}_i - \sum_{i=1}^{n} VAR_i) \Big/ \sum_{i=1}^{n} VBAR_i] \times 100$$

and $\sum_{i=1}^{n} \hat{VBAR}_i$ and $\sum_{i=1}^{n} VBAR_i$ are the sum of the predicted and actual VBARs, respectively.

For the validation data set, average DBH=10.8 in. (range: 8.4 to 15.3), average merchantable height=2.8 sticks (range: 1 to 6), average cubic-foot VBAR= 17.24 cu.ft./sq.ft. (range: 6.58 to 33.18), average Doyle board-foot VBAR=52.68

Table 8. Regression equations for predicting cubic-foot VBARs (CR) and Doyle (DR), International 1/4-inch (IR), and Scribner (SR) board-foot VBARs from the other three types of VBARs.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CR} = 1.8823+0.3582DR-0.000957DR^2$	0.977	1.06
$\hat{CR} = 0.7074+0.1989IR-0.000170IR^2$	0.994	0.53
$\hat{CR} = 0.8457+0.2131SR-0.000223SR^2$	0.993	0.59
$\hat{DR} = 2.3011+1.4972CR+0.07034CR^2$	0.972	4.88
$\hat{DR} = 0.5795+0.4264IR+0.001244IR^2$	0.987	3.36
$\hat{DR} = 0.3878+0.4778SR+0.001172SR^2$	0.991	2.82
$\hat{IR} = -1.2647+4.6274CR+0.03850CR^2$	0.994	3.60
$\hat{IR} = 4.1983+1.9308DR-0.003807DR^2$	0.988	5.06
$\hat{IR} = 0.4153+1.0835SR-0.000254SR^2$	1.000	0.83
$\hat{SR} = -1.0245+4.1675CR+0.04537CR^2$	0.993	3.54
$\hat{SR} = 3.0566+1.8112DR-0.003221DR^2$	0.992	3.73
$\hat{SR} = -0.3032+0.9201IR+0.000228IR^2$	1.000	0.73

bd.ft./sq.ft. (range: 15.16 to 129.65), average International 1/4-inch board-foot VBAR=92.02 bd.ft./sq.ft. (range: 30.11 to 191.76), and average Scribner board-foot VBAR=86.71 bd.ft./sq.ft. (range: 27.90 to 184.32).

Table 9 shows $\overline{RE}$, range of RE_i, and RE_S for the cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBAR prediction equations based on height only (Equations 1-4) and height and DBH (Equations 5-8) independent variables. $\overline{RE}$ was 1.3% or less and RE_S was 0.33% or less for all VBAR prediction equations. Plots of RE_i versus $VBAR_i$ showed no prediction problems for the range of $VBAR_i$s in the validation data set with a slight tendency to overpredict actual values, especially for smaller $VBAR_i$s. There was relatively little difference between the height only and height and diameter prediction models in terms of accuracy of predictions.

Table 9. Average relative error ($\overline{RE}$), range of RE_i, and relative error for the sum of the predicted values (RE_S) for the four VBAR prediction equations based on height only independent variables (Equations 1-4) and height and diameter independent variables (Equations 5-8).

Prediction Equation		$\overline{RE}$	Range of RE_i	RE_S
Cubic-foot	(1)	0.7	-15.4 to 19.2	0.33
	(5)	0.6	-16.2 to 18.4	0.29
Doyle	(2)	1.3	-23.7 to 25.3	0.17
	(6)	1.3	-19.8 to 25.5	0.23
International 1/4-inch	(3)	0.7	-14.6 to 26.9	0.23
	(7)	0.7	-17.9 to 16.9	0.23
Scribner	(4)	0.8	-15.3 to 25.5	0.23
	(8)	0.8	-18.0 to 18.2	0.24

Guidelines for Users

The new cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBAR prediction equations using DBH and height independent variables were somewhat more accurate than the respective equations using only height independent variables (Tables 1, 3 and 9). However, since the differences in accuracy between the 2 sets of equations were very small, the simpler height only VBAR equations are more than adequate for most situations.

The new International 1/4-inch board-foot VBAR equation would yield per acre estimates from 14.3% lower (all trees with 1 stick) to 0.55% higher (all trees with 6 sticks) than estimates based on the VBARs presently used by the DNR. In general, estimates using our VBARs would be no more than 3% less than estimates using DNR VBARs except for stands having many trees with only one sawtimber stick.

We recommend the use of VBAR Equations 1-4 for most cruising situations. For those situations where somewhat more accuracy is needed and DBH is already measured for some other purpose(s), Equations 5-8 may be used.

If the user has access to a set of VBAR equations for just one type of volume, VBARs for other types of volume can be predicted from the appropriate equation in Table 8.

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