INDIVIDUAL TREE SAWTIMBER VOLUME EQUATIONS FOR RED PINE IN MICHIGAN

BY

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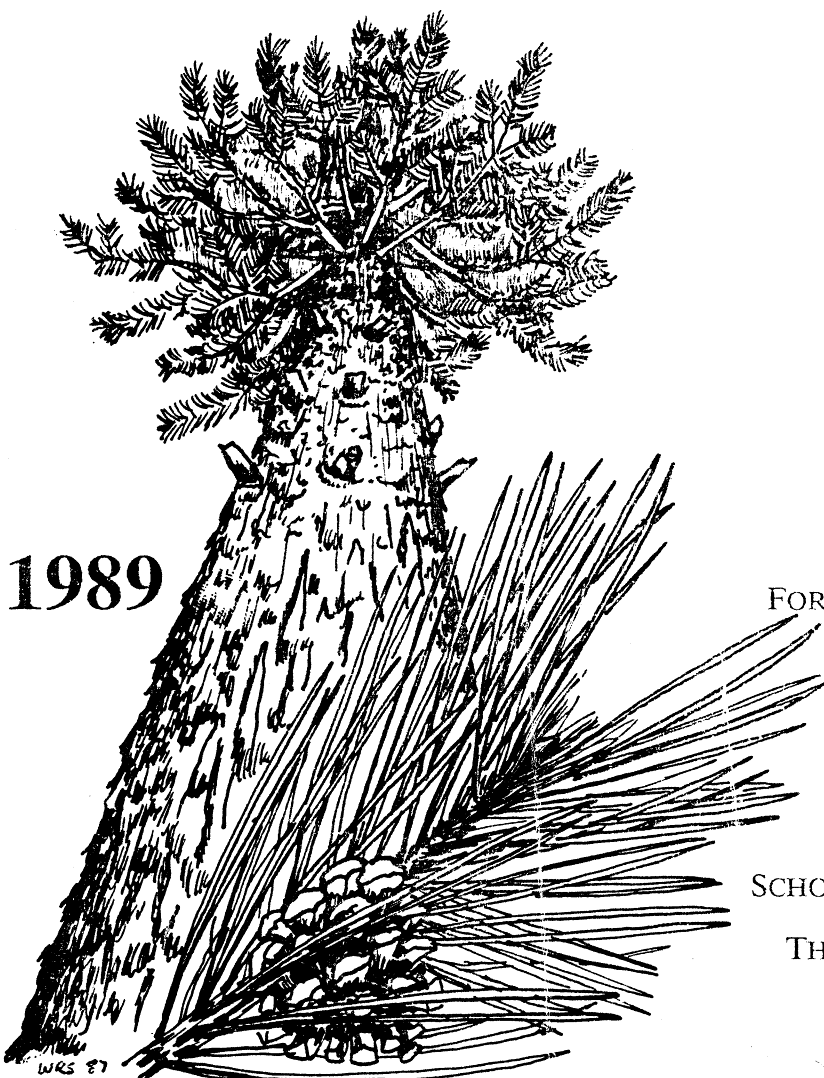

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FOREST MANAGEMENT DIVISION
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MANAGEMENT SUMMARY

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot individual tree sawtimber volume equations were developed for red pine in Michigan. Data used to develop these equations were collected from 27 red pine stands in Michigan (16 and 11 stands from the Upper and Lower Peninsulas, respectively).

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y.x}$), and an independent validation data set for a series of linear and nonlinear regression equations indicated that nonlinear equations were most accurate for all types of volume. The new International 1/4-inch and Scribner board-foot volume equations yielded volume estimates smaller than the values in Tables 1 and 2 of Gevorkiantz and Olsen (1955) for DBHs $\geq$ 21 in. and $\geq$ 24 in., respectively, with mixed results for smaller DBHs. The new individual tree volume equations are:

1. Cubic-foot volume

$$\hat{V}_c = 0.1035D^{1.614}H^{0.8688}$$

2. Doyle board-foot volume

$$\hat{V}_d = 0.06032D^{2.280}H^{0.9635}$$

3. International 1/8-inch board-foot volume

$$\hat{V}_I = 0.4050D^{1.753}H^{0.9497}$$

4. Scribner board-foot volume

$$\hat{V}_s = 0.3034D^{1.807}H^{0.9542}$$

where D is DBH in inches and H is merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit. International 1/4-inch volumes are obtained by multiplying 0.905 times volumes obtained from Equation 3.

Multiple linear regression equations were developed to predict (1) one type of volume from another type and (2) Doyle, International 1/4-inch, and Scribner board-foot cubic-foot ratios as a function of D, H, and D and H.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - INDIVIDUAL TREE VOLUME EQUATIONS

DATE - 16 December 88

TITLE - Individual Tree Sawtimber Volume Equations for Red Pine in Michigan

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Background

Composite individual tree sawtimber board-foot (i.e., International 1/4 and Scribner) volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955). The tables of Gevorkiantz and Olsen or some modifications of them are still widely used in Michigan for red pine.

Purpose

The purpose of this paper is to present new sawtimber cubic-foot and Doyle, International 1/4-inch, and Scribner volume equations and tables for red pine in Michigan.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 3507 trees from 27 stands as follows:

1) 2341 trees from 16 stands in the Upper Peninsula (i.e., 5, 4, and 7 stands each in the eastern, central, and western U.P., respectively), and
2) 1166 trees from 11 stands in the Lower Peninsula (i.e., 4, 4, and 3 stands from the northeast, northwest, and southern L.P., respectively.)

Stands were selected from the above six regions to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May - August 1983-1985.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 years, basal area/acre varied from 90 to 225 sq. ft., average DBH varied form 6.7 to 17.7 in., average total height varied from 33.2 to 86.0 ft., and average merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter varied from 2.0 to 8.4 100-in. sticks. For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b) developed pulpwood, sawtimber, and residual pulpwood cubic-foot volume equations from the total data set described above.

Sawtimber trees were defined as trees that had at least one 100-in. stick with a minimum inside bark diameter no smaller than 7.6 inches. Sawtimber merchantable height (MH) is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no larger than 7.6 inches. There was a total of 2,157 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. For each 100-in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V = \frac{(B+b)L}{2}$ (Avery and Burkhart, 1983)

Doyle: $V = 0.5D^2 - 4.0D + 8.0$ (Husch et al. 1982)

International 1/4-inch: $V = 0.905(0.44D^2 - 1.20D - 0.30)$ (Husch et al. 1982)

Scribner: $V = 0.395D^2 - 0.99D - 2.15$ (Bruce and Schumacher 1950)

where

V = volume in cubic feet or board feet,

L = length of stick (100 in.) in ft.,

B = cross-sectional area inside bark of large end of the stick in sq. ft.,

b = cross-sectional area inside bark of small end of stick in sq. ft., and

D = diameter of small end of stick inside bark in inches.

See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the three board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 7.6-in. top diameter limit. International 1/8-inch volumes were calculated for each tree, and all regression equations were developed using these volumes. International 1/4-inch volumes were obtained by multiplying 0.905 times the International 1/8-inch volumes.

Individual tree volume was regressed on various variables determined from tree DBH and MH using multiple linear and nonlinear regression.

Results

The data set used to develop the regression equations consisted of 1725 trees (≈80%) selected at random from the total of 2157 trees. Fowler and Hussain (1987b) found no significant differences between the pulpwood cubic-foot volume equations of the six regions, so the data from all stands were pooled before developing volume regression equations. All equations were based on 1725 trees with an average DBH = 12.2 in. (range: 7.9 to 23.9), average MH = 3.7 sticks (range: 1 to 8), average cubic-foot volume = 21.23 (range: 2.78 to 99.56), average Doyle board-foot volume = 85.3 (range: 5.0 to 554.2), average International 1/8-inch volume = 136.5 (range: 13.7 to 711.0), and average Scribner volume = 119.0 (range: 11.1 to 633.8).

Cubic-foot and Board-foot Volume Prediction Equations

A comparison of various multiple linear regression and nonlinear regression equations based on goodness-of-fit and simplicity indicated that the following nonlinear prediction equation compared favorably to all other equations examined for cubic-foot and the three board-foot volumes:

$$\hat{V} = \hat{\beta}_0 D^{\hat{\beta}_1} H^{\hat{\beta}_2}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is MH in 100 in. sticks to a 7.6-in. top diameter limit. $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the sample regression coefficients related to the independent variables.

Table 1 shows the sawtimber volume prediction equations for cubic-foot and Doyle, International 1/8-inch, and Scribner board-foot volumes along with coefficients of variation (R^2) and standard errors of the estimate ($s_{y \cdot x}$).

A cubic-foot volume table is shown in Table 2, and Doyle, International 1/4-inch, and Scribner board-foot volume tables are shown in Tables 3, 4, and 5, respectively. The International 1/4-inch board-foot values in Table 4 were obtained by multiplying 0.905 times the values of Equation 3.

The values in our Table 4 (International 1/4-inch board-foot volumes) are smaller for lower MHs and larger for higher MHs compared to the values in Table 2 of Gevorkiantz and Olsen (1955) for DBHs less than 21 inches. The MH where the values in our table become larger increases as DBH increases. For DBHs of 21 inches and over, our table, in general, gives smaller values. A similar comparison holds between values in our Table 5 (Scribner board-foot volume) and values in Table 1 of Gevorkiantz and Olsen (1955) except that the cutoff DBH is 24 inches instead of 21 inches.

Table 1. Estimated parameters ($\hat{B}_0$, $\hat{B}_1$, and $\hat{B}_2$), and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and three board-foot volumes.

Prediction Equation	$\hat{B}_0$	$\hat{B}_1$	$\hat{B}_2$	R^2	$s_{y \cdot x}$
(1) Cubic-foot[a]	0.1035	1.614	0.8688	0.991	1.64
(2) Doyle[b]	0.06032	2.280	0.9635	0.978	13.50
(3) International 1/8-inch[c]	0.4050	1.753	0.9497	0.984	15.59
(4) Scribner[d]	0.3034	1.807	0.9542	0.983	14.16

[a] $\hat{V}=0.1035D^{1.614}H^{0.8688}$

[b] $\hat{V}=0.06032D^{2.280}H^{0.9635}$

[c] $\hat{V}=0.4050D^{1.753}H^{0.9497}$

[d] $\hat{V}=0.3034D^{1.807}H^{0.9542}$

Table 2. Volume table showing cu. ft. volume for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 1).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	3.6	6.6	9.3	12.0	14.5				
10	4.3	7.8	11.1	14.2	17.2	20.2			
11	5.0	9.1	12.9	16.6	20.1	23.5			
12	5.7	10.4	14.8	19.0	23.1	27.1	31.0		
13	6.5	11.9	16.9	21.7	26.3	30.8	35.2	39.6	
14	7.3	13.4	19.0	24.4	29.7	34.7	39.7	44.6	
15	8.2	15.0	21.3	27.3	33.1	38.8	44.4	49.9	55.2
16		16.6	23.6	30.3	36.8	43.1	49.3	55.3	61.3
17		18.3	26.0	33.4	40.6	47.5	54.3	61.0	67.6
18		20.1	28.5	36.6	44.5	52.1	59.6	66.9	74.1
19		21.9	31.1	40.0	48.5	56.9	65.0	73.0	80.9
20		23.8	33.8	43.4	52.7	61.8	70.6	79.3	87.9
21			36.6	47.0	57.1	66.8	76.4	85.8	95.1
22			39.5	50.7	61.5	72.1	82.4	92.5	102.5
23			42.4	54.4	66.1	77.4	88.5	99.4	110.1
24			45.4	58.3	70.8	82.9	94.8	106.5	117.9
25			48.5	62.3	75.6	88.6	101.3	113.7	126.0

Table 3. Volume table showing Doyle bd. ft. volume for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 2).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	9	18	26	34	43				
10	11	22	33	44	54	65			
11	14	28	41	54	67	80			
12	17	34	50	66	82	98	114		
13	21	41	60	79	99	117	136	155	
14	25	48	71	94	117	139	161	184	
15	29	56	83	110	137	163	189	215	241
16		65	97	128	158	189	219	249	279
17		75	111	147	182	217	251	286	320
18		86	127	167	207	247	286	326	365
19		97	143	189	234	279	324	368	413
20		109	161	212	263	314	364	414	464
21			180	237	294	351	407	463	518
22			200	264	327	390	452	514	576
23			221	292	362	431	501	569	638
24			244	322	399	475	552	627	703
25			268	353	438	522	605	688	771

Table 4. Volume table showing International 1/4-inch bd. ft. volume for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 3).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	17	33	49	64	80				
10	21	40	59	77	96	114			
11	25	47	70	92	113	134			
12	29	55	81	107	132	157	181		
13	33	63	93	123	152	180	209	237	
14	37	72	106	140	173	205	238	270	
15	42	82	120	158	195	232	268	304	340
16		91	134	176	218	259	300	341	381
17		102	149	196	243	288	334	379	424
18		112	165	217	268	319	369	419	469
19		123	182	239	295	351	406	461	515
20		135	199	261	323	384	444	504	564
21			216	284	351	418	484	549	614
22			235	308	381	453	525	596	666
23			254	333	412	490	567	644	720
24			273	359	444	528	611	694	776
25			294	386	477	567	657	745	834

Table 5. Volume table showing Scribner bd. ft. volume for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 4).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	16	31	46	60	75				
10	19	38	55	73	90	108			
11	23	45	66	87	107	128			
12	27	52	77	102	126	149	173		
13	31	61	89	117	145	173	200	227	
14	36	69	102	134	166	198	229	260	
15	40	78	115	152	188	224	259	294	329
16		88	130	171	211	251	291	331	370
17		98	145	191	236	281	325	369	413
18		109	161	211	261	311	360	409	458
19		120	177	233	288	343	397	451	505
20		132	194	256	316	376	436	495	554
21			212	279	345	411	476	541	605
22			231	304	376	447	518	588	658
23			250	329	407	484	561	637	713
24			270	355	440	523	606	688	770
25			291	382	473	563	652	741	829

In comparing the two sets of tables, it should be kept in mind that our tables assume a 0.5 ft. stump, a minimum top diameter as close as possible to but no smaller than 7.6 inches, and log lengths of 8 ft., whereas the tables of Gevorkiantz and Olsen assume a 1.0-ft. stump, a minimum top diameter that is variable, is no smaller than 8.0 in., and can be considerably larger than 8.0 inches, and log lengths of 16 ft.

Predicting One Type of Volume from Another

Multiple linear regression equations were developed to predict one type of volume from another using the 1725 trees in the prediction data set. Equations were developed for predicting cubic-foot volume (CV) as a function of Doyle (DV), International 1/4-inch (IV), and Scribner (SV) board-foot volume, DV as a function of CV, IV, and SV, IV as a function of CV, DV, and SV, and SV as a function of CV, DV, and IV. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 6.

Predicting Board-foot Cubic-foot ratios

Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios were calculated for each of the 1725 trees in the prediction data set. Average board-foot cubic-foot ratios were 3.37 (range: 1.32 to 5.80), 5.50 (range: 3.26 to 6.74) and 5.22 (range: 2.90 to 6.64) for Doyle, International 1/4-inch, and Scribner board-foot volumes, respectively.

Multiple linear regression equations were developed to predict the three types of board-foot cubic-foot ratios as a

Table 6. Regression equations for predicting cubic-foot (CV) and Doyle (DV), International 1/4-inch (IV), and Scribner (SV) board-foot volumes from the other three types of volumes.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CV} = 2.8386 + 0.2464\ DV - 0.000170\ DV^2$	0.993	1.42
$\hat{CV} = 0.8073 + 0.1745\ IV - 0.000042\ IV^2$	0.999	0.59
$\hat{CV} = 1.0744 + 0.1804\ SV - 0.000051\ SV^2$	0.999	0.65
$\hat{DV} = -6.7966 + 3.3643\ CV + 0.027469\ CV^2$	0.994	6.85
$\hat{DV} = -5.5134 + 0.6369\ IV + 0.000442\ IV^2$	0.996	5.77
$\hat{DV} = -4.7617 + 0.6667\ SV + 0.000414\ SV^2$	0.997	5.11
$\hat{IV} = -3.9895 + 5.6233\ CV + 0.010876\ CV^2$	0.999	3.83
$\hat{IV} = 10.9127 + 1.4531\ DV - 0.000731\ DV^2$	0.996	7.26
$\hat{IV} = 1.4459 + 1.0372\ SV - 0.000051\ SV^2$	1.000	0.86
$\hat{SV} = -5.0485 + 5.3911\ CV + 0.012768\ CV^2$	0.999	4.05
$\hat{SV} = 8.9397 + 1.4092\ DV - 0.000656\ DV^2$	0.997	6.28
$\hat{SV} = -1.3673 + 0.9634\ IV + 0.000049\ IV^2$	1.000	0.84

function of D, H, and D and H. These equations and their associated values of R^2 and $s_{y \cdot x}$ are shown in Table 7.

Board-foot cubic-foot ratios for the three types of board-foot volume as a function of H and D are shown in Tables 8 and 9, respectively.

<u>Validation</u>

The data set used to validate the prediction equations consisted of 432 trees, the remaining approximately 20% of the total of 2157 trees. For each volume equation, the average relative error as a percent ($\overline{RE}$) was calculated where

$$\overline{RE} = \sum_{i=1}^{n} RE_{i/n}$$

and $RE_i = [(\hat{V}_i - V_i)/V_i] \times 100$, $\hat{V}_i$ and V_i are the predicted and actual volumes for the i^{th} tree, and n is the number of trees in the validation data set (n=432). The relative error as a percent for the sum of the predicted volumes was also calculated where

$$RE_s = [(\sum_{i=1}^{n} \hat{V}_i - \sum_{i=1}^{n} V_i)/ \sum_{i=1}^{n} V_i)] \times 100$$

and $\sum_{i=1}^{n} \hat{V}_i$ and $\sum_{i=1}^{n} V_i$ are the sum of the predicted and actual volumes, respectively.

For the validation data set, average DBH = 12.4 in. (range: 2.9 to 22.7), average MH = 3.7 sticks (range: 1 to 9), average cubic-foot volume = 21.8 cu. ft. (range: 2.8 to 88.5), average Doyle board-foot volume = 87.6 bd. ft. (range: 6.6 to 476.8),

Table 7. Regression equations for predicting Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D, H, and D and H.

Type of Ratio	Regression Equation	R^2	$s_{y \cdot x}$
Doyle	$\hat{DCR} = 0.342 + 0.248D$	0.905	0.26
	$\hat{DCR} = 1.952 + 0.388H$	0.725	0.44
	$\hat{DCR} = 0.569 + 0.192D + 0.123H$	0.933	0.22
International 1/4-inch	$\hat{ICR} = 4.082 + 0.116D$	0.659	0.27
	$\hat{IRC} = 4.706 + 0.216H$	0.753	0.23
	$\hat{ICR} = 4.359 + 0.048D + 0.150H$	0.797	0.21
Scribner	$\hat{SCR} = 3.475 + 0.143D$	0.725	0.28
	$\hat{SCR} = 4.281 + 0.256H$	0.766	0.26
	$\hat{SCR} = 3.766 + 0.072D + 0.158H$	0.836	0.22

Table 8. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of H.

H	Board-foot Cubic-foot Ratio		
Sticks	DCR	ICR	SCR
1	2.3	4.9	4.5
2	2.7	5.1	4.8
3	3.1	5.4	5.0
4	3.5	5.6	5.3
5	3.9	5.8	5.6
6	4.3	6.0	5.8
7	4.7	6.2	6.1
8	5.1	6.4	6.3
9	5.4	6.6	6.6
10	5.8	6.9	6.8

Table 9. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D.

D	Board-foot Cubic-foot Ratio		
(in.)	DCR	ICR	SCR
9	2.6	5.1	4.8
10	2.8	5.2	4.9
11	3.1	5.4	5.0
12	3.3	5.5	5.2
13	3.6	5.6	5.3
14	3.8	5.7	5.5
15	4.1	5.8	5.6
16	4.3	5.9	5.8
17	4.6	6.1	5.9
18	4.8	6.2	6.0
19	5.1	6.3	6.2
20	5.3	6.4	6.3
21	5.6	6.5	6.5
22	5.8	6.6	6.6
23	6.0	6.8	6.8
24	6.3	6.9	6.9
25	6.5	7.0	7.0

average International 1/4-inch volume = 126.7 bd. ft. (range: 14.6 to 566.4), and average Scribner board-foot volume = 122.1 bd. ft. (range: 13.3 to 556.7).

Table 10 shows $\overline{RE}$, range of RE_i, and RE_s for the cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot prediction equations.

$\overline{RE}$ was less than 4.5% and RE_s was less than 2.5% for all volume prediction equations. Plots of RE_i versus V_i showed no prediction problems for the range of V_i's in the validation data set except for small values of V_i. The volume prediction equations somewhat over-predict volume for cubic-foot volumes less than 5 and Doyle, International 1/4-inch and Scribner board-foot volumes less than 30, 30, and 20, respectively.

Guidelines for Users

We recommend the use of Equations 1-4 for estimating individual tree sawtimber volume for red pine in Michigan for most cruising situations. The merchantability limits assumed by these equations closely approximate merchantability limits used by the Michigan Department of Natural Resources.

If the user wants to predict one type of volume from another, the appropriate equation in Table 6 can be used. If the user needs to estimate board-foot cubic-foot ratios for one of the types of board-foot volumes, the appropriate equation in Table 7 can be used.

Table 10. Average relative error ($\overline{RE}$), range of RE_i, and relative error for the sum of the predicted values (RE_s) for the four volume prediction equations (Equations 1-4).

Prediction Equation	$\overline{RE}$	Range of RE_i	RE_s
(1) Cubic-foot	1.2	-19.2 to 16.2	1.1
(2) Doyle	4.3	-32.1 to 41.0	2.4
(3) International 1/8-inch	1.1	-25.7 to 24.8	1.4
(4) Scribner	1.5	-26.3 to 26.7	1.4

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MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES FOREST MANAGEMENT DIVISION