Manual of Fisheries Survey Methods II: with periodic updates

Chapter 12: Three Methods for Computing the Volume of a Lake

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Chapter 12: Three Methods for Computing the Volume of a Lake

Clarence M. Taube

[Editor's note: Area of lakes, a basic requirement for volume calculations, has traditionally been estimated with a mechanical planimeter. In recent years, computer programs can compute area (such as Arcview™) and volume (such as Tecplot®).]

12.1 Method No. 1

The formula in solid geometry for calculating the volume of a frustum of a circular cone has been applied by limnologists and fisheries biologists to compute the volume of a lake. This formula is:

$$V = \tfrac{1}{3} H\left(A_1 + A_2 + \sqrt{A_1 \times A_2}\right)$$

Where: V = volume of water;

H = difference in depth between two successive depth contours;

A_1 = area of the lake within the outer depth contour being considered;

A_2 = area of the lake within the inner contour line under consideration.

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The procedure consists of determining the volumes of successive layers of water (frustums), and then summing these volumes to obtain the total volume of the lake.

12.2 Method No. 2

Another formula has occasionally been used for computing lake volume. This method is employed by engineers for computing reservoir volumes, and is derived from the "end-area formula" sometimes applied to find the volume of prismoidal forms. The formula is:

$$V = \tfrac{1}{2} H\left(A_1 + A_2\right)$$

Variables and general procedures are the same as in Method No. 1.

12.3 Method No. 3

A third method for estimating lake volume is to determine average lake depth and multiply it by lake area. Average depth is obtained by averaging depth soundings. For a reliable average, the soundings should be spaced in a uniform grid pattern. The accuracy of this method depends on frequent soundings at regular intervals and the recording of all soundings. The omission of depth soundings for very shallow water (e.g., close to shore) is a common source of error in the application of this method.

12.4 Procedures

The first two methods require an accurate depth contour map and a planimeter. The third method requires a field map showing actual depths (on a grid pattern) and a planimeter.

When working with either Method No. 1 or No. 2, first determine area within each contour by tracing around map contour lines with a planimeter. Start with the shoreline contour and continue to the innermost contour line. The resultant readings will be in square inches (the unit of measure of the planimeter). Then, based on the scale of the field map, convert planimeter readings to values of lake area, either in acres or square feet. For very small ponds it may be desirable to compute areas in square feet, but lake area is commonly expressed in acres.

As an example of calculating lake area, and then areas within consecutive contours, assume that a lake map was drawn on the scale of 1 inch equals 100 feet. Then, 1-square inch of map area (planimeter reading) equals 10,000 square feet (100 × 100) of lake area, or 0.22957 acres (10,000/43,560). Further assume for this example that the lake has a maximum depth of 23 feet, depth contours were drawn for each 5-foot interval, and planimeter readings for the area within the contours were as in the following table:

Depth contour (feet)	Planimeter reading (square inches)	Calculated area	
		Square feet	Acres
(Shoreline contour) 0	210.0	2,100,000	48.2
5	150.0	1,500,000	34.4
10	110.0	1,100,000	25.3
15	83.5	835,000	19.2
20	21.7	217,000	5.0
(Maximum depth) 23			

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Those calculated areas were obtained by conversion factors of 10,000 for square feet and 0.22957 for acres. Area in acres could have been calculated by dividing area in square feet by 43,560 (number of square feet per acre). Note that 48.2 acres is the calculated total area of the lake.

As an illustration of computing water volumes, we continue to use the sample data given above. By Method No. 1, calculations of volume (in acre-feet) are as follows:

Depth strata	Equation	Acre feet
0 - 5 ft :	$\frac{1}{3} * 5\left(48.2 + 34.4 + \sqrt{48.2 * 34.4}\right)$	= 205.5
5 - 10 ft :	$\frac{1}{3} * 5\left(34.4 + 25.3 + \sqrt{34.4 * 25.3}\right)$	= 148.7
10 - 15 ft :	$\frac{1}{3} * 5\left(25.3 + 19.2 + \sqrt{25.3 * 19.2}\right)$	= 110.8
15 - 20 ft :	$\frac{1}{3} * 5\left(19.2 + 5.0 + \sqrt{19.2 * 5.0}\right)$	= 56.7
20 - 23 ft :	$\frac{1}{3} * 3\left(5.0\right)$	= 5.0
	Total volume	= 526.7

Note that the volume calculation for the lowermost layer (20-23 ft) uses the cone formula: volume = 1/3(HA). By applying the formula here we assume that the maximum depth of 23 feet occurred only in a small area. If the maximum depth of 23 feet prevailed over an extensive area, then encircle this area with a contour line, determine its area with a planimeter, and use the frustum formula to calculate volume of the 20-23-foot zone.

By Method No. 2, example calculations of volume (in acre-feet) are as follows:

Depth strata	Equation		Acre feet
0 - 5 ft :	$\frac{1}{2} * 5(48.2 + 34.4)$	=	206.5
5 - 10 ft :	$\frac{1}{2} * 5(34.4 + 25.3)$	=	149.3
10 - 15 ft :	$\frac{1}{2} * 5(25.3 + 19.2)$	=	111.3
15 - 20 ft :	$\frac{1}{2} * 5(19.2 + 5.0)$	=	60.5
20 - 23 ft :	$\frac{1}{2} * 3(5.0)$	=	7.5
	Total volume	=	535.1

When using acres for area values and feet for depth values, volume will be in acre-feet. An acre-foot of water is 1 acre of water 1 foot deep, i.e., 43,560 cubic feet.

In Method No. 3, all soundings of the lake are summed, then divided by the number of soundings to obtain average depth. Lake volume equals average depth times lake area. Lake area is determined by planimeter measurements on the field map, as described above.

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12.5 Comparison of the three methods

The three methods give approximate rather than exact volumes of lakes, but these approximations are close enough to the true values for fisheries work. These methods usually give quite similar results, as demonstrated by the three example lakes below:

Name of lake	Area (acres)	Location	Computed volumes (acre-feet) Method No.1	Method No.2	Method No.3
Frost	60.0	Ogemaw Co.	1,949	1,963	1,977
Robinson	20.3	Oakland Co.	64	63	58
Eagle	19.9	Oakland Co.	137	142	138

However, based on years of experience, Methods 2 and 3 often give slightly higher values than Method 1 (the Frost Lake example is one of the exceptions). The slight difference appears to be related to lake basin shape, but has no practical significance to fisheries work. Methods Nos. 2 and 3 are preferable to Method No. 1 from the standpoint of simplicity. Assuming that the lake in question is shaped like a series of frustums, the formula of Method No. 1 is mathematically correct.

Written 4/1947 by C. M. Taube
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