An Aspen Bark Factor Equation for Michigan

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ABSTRACT. A multiple linear regression equation was developed to predict bark factor for aspen in Michigan as a function of tree height. Bark factors for bigtooth aspen were, in general, somewhat larger than bark factors for trembling aspen. Even though equations were developed for both species, the differences between the two equations were small, and not statistically significant, and a pooled equation based on both species is recommended. The pooled prediction equation yielded average relative errors from −2.3 to 0.87% and −1.02 to 3.83% at all tree heights for bigtooth and trembling aspen, respectively. For more accurate predictions of bark factor, the separate prediction equations for bigtooth and trembling aspen should be used. The new equations can be used to more accurately estimate tree and log wood volumes than when using a constant bark factor determined at breast height, which, in general, leads to underestimates of wood volumes.

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Bark factor (F) at a given tree height is the ratio of diameter inside bark to diameter outside bark. Bark factors vary with species, age, site, and tree height. Bark factors at stump or breast height usually vary from 0.87 to 0.93. Even though much of the variation in bark factor is related to species, bark factor does increase with tree height for many species. In spite of this relationship, a constant bark factor has been assumed for many tree species for all tree heights. The use of a constant bark factor, determined at breast height, for all tree heights, will, in general, lead to underestimates of most tree and log volumes and overestimates of bark volume.

Multiple linear regression equations have been developed to predict bark factor as a function of various independent variables such as tree height and associated diameter outside bark. Such equations have not been developed for many species because data have been lacking for the independent variables, or the use of a constant bark factor has been considered adequate. As forest management becomes more intensive, the use of such equations should be considered so that more accurate estimates of wood volume can be obtained. Wood values should be estimated as accurately as possible in order to more accurately assess timber values in multiple-use forest management. See Husch et al. (1982) for a detailed general discussion on bark factors, and specifically refer to Fowler and Damschroder (1988) who developed a red pine bark factor equation for Michigan and discussed the various uses of bark factors.

The objective of this study was to develop a bark factor prediction equation for aspen, both bigtooth and trembling, in Michigan.

PROCEDURES

The data set used to develop the prediction equation consisted of felled tree measurements on a total of 302 aspen (181 bigtooth and 121 trembling) trees from 5 aspen stands in Michigan (2 stands in each of the Pere Marquette and Mackinaw State Forests and 1 in the Escanaba River State Forest). Diameter inside and outside bark were measured to the nearest 0.1 in. at stump height and at the top of each 8.3-ft (100-in.) stick cut out of each tree to an approximate 3.6 in. diameter top limit. Dbh was measured to the nearest 0.1 in. with a D-tape, and the bark thickness to the nearest 0.05 in. at 4.5 ft above the ground was determined using a hatchet and a ruler. The number of trees and average and range of dbh in inches and merchantable height in 8.3-ft bolts are shown in Table 1.

The data set used to validate the prediction equation consisted of 127 aspen (72 trembling and 55 bigtooth) trees from 3 aspen stands in Michigan (1 each in the Au Sable, Lake Superior, and Mackinaw State Forests). The number of trees and average, minimum, and maximum values of dbh in inches and merchantable height in feet are shown in Table 2. Merchantable heights are given in feet because variable bolt lengths were cut from 2 of these stands. The same measurements were made on these trees as were made on the prediction data set trees.

For the prediction data set, the bark factor at each tree height was determined using all of the trees with measurements at that height with the formula

$$k = \frac{\text{sum of diameters inside bark}}{\text{sum of diameters outside bark}}$$

A good discussion on equations to determine bark factor is presented in Husch et al. (1982).

RESULTS AND DISCUSSION

The variation of average k at a given height among stands in the Escanaba River, Mackinaw, and Pere Marquette State Forests was relatively small, justifying pooling of all data at a given height for each species (Table 3). For a given height, average k's for the 5 stands with bigtooth aspen were within 0.012 to 0.031 of each other while average k's for the 4 stands with trembling aspen were within 0.007 to 0.024 of each other. The larger variations occurred at larger heights where numbers of sample trees were small.

For all heights, average k for bigtooth aspen was larger than k for trembling aspen, the difference varying from 0.0001 to 0.031, with very small differences for lower heights and larger differences for larger heights. When both species were pooled, average k was 0.9130 and 0.9302 at 0.33+ and 4.5 ft, respectively, increased to 0.9314 at 8.7 ft, and decreased to 0.8913 at 67.0 ft.

Bark factor was plotted against tree height for bigtooth aspen, trembling aspen, and both species pooled, indicating that bark factor (Y) would be very closely predicted by some combination of the following forms of tree height (X): X, $1/X$, and $\ln X$. A set of prediction equations (i.e., all combinations of X, $1/X$, and $\ln X$) was constructed using weighted multiple linear regression with weights based on the number of trees with measurements at that height for 10 heights (Table 3). The best prediction equation, i.e., that equation that yielded the smallest standard errors of the regression coefficients ($S_{\hat{\beta}_0}$, $S_{\hat{\beta}_1}$ and $S_{\hat{\beta}_2}$) and the largest coefficient of multiple determination (R^2), was

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 \ln X$$

where $\hat{Y}$ is estimated k and X is tree height in feet. The prediction equations for bigtooth aspen, trembling aspen, and both species pooled along with $S_{\hat{\beta}_0}$, $S_{\hat{\beta}_1}$ and $S_{\hat{\beta}_2}$ and R^2 are found in Table 4. All 3 regression equations were highly significant ($P < 0.001$). Skewness and kurtosis coefficients of the residuals were $< \pm 1$ for the trembling aspen and pooled equations, and 1.1 and 1.9, respectively, for the bigtooth aspen equation, indicating, in general, no serious departures from normality given only $n = 10$ tree

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Table 1. Number of trees, average ($\bar{x}$) and minimum and maximum values of dbh in inches and merchantable height (m.ht.) in 100-in. sticks for the 5 data sets used to construct the prediction equation.

Region	State forest	Stand	Aspen species	No. of trees	dbh $\bar{x}$	dbh Min.–Max.	m.ht. $\bar{x}$	m.ht. Min.–Max.
U.P.	Escanaba River	1	BT	16	11.7	8.4–16.2	7.0	6–8
			T	44	11.0	8.4–14.6	6.5	5–8
L.P.	Mackinaw	1	BT	40	8.4	4.7–11.9	4.7	2–6
			T	20	8.2	6.1–12.0	4.1	3–6
		2	BT	40	8.9	5.3–12.5	5.0	2–7
			T	20	8.1	6.4–10.2	4.2	3–5
	Pere Marquette	1	BT	24	8.2	4.6–12.8	4.1	3–6
			T	37	7.4	4.6–10.3	3.8	2–5
		2	BT	61	7.5	5.1–14.0	4.0	2–6

Table 2. Number of trees, average ($\bar{x}$) and minimum and maximum values of dbh in inches and merchantable height (m.ht.) in ft for the 3 data sets used to validate the prediction equations.

Region	State forest	Stand	Aspen species	No. of trees	dbh $\bar{x}$	dbh Min.–Max.	m.ht. $\bar{x}$	m.ht. Min.–Max.
U.P.	Lake Superior	1	BT	16	12.4	8.1–15.1	53.7	25.2–61.4
			T	38	10.3	7.0–13.3	47.7	34.0–60.6
L.P.	Au Sable	1	BT	4	11.1	8.5–14.4	43.7	33.3–50.0
			T	27	9.1	6.6–12.6	41.1	25.0–58.3
	Mackinaw	1	BT	35	11.7	7.7–16.6	65.1	50.5–73.5
			T	7	11.4	8.9–13.5	50.9	42.5–68.2

Note: Bolt length for the Au Sable stand was 8.3 ft, and bolt lengths for the Lake Superior and Mackinaw stands varied from 8.3 to 9.1 ft and 7.8 to 11.4 ft, respectively.

Table 3. Number of trees, bark factors, and predicted bark factors for bigtooth aspen (BT), trembling aspen (T), and both species pooled for 10 tree heights in feet.

Tree height	No. of trees BT	No. of trees T	No. of trees Pooled	Observed bark factor BT	Observed bark factor T	Observed bark factor Pooled	Predicted bark factor BT	Predicted bark factor T	Predicted bark factor Pooled
0.33+	181	121	302	0.9134	0.9124	0.9130	0.9134	0.9128	0.9131
4.5	181	121	302	0.9301	0.9300	0.9300	0.9302	0.9290	0.9297
8.7	181	121	302	0.9317	0.9310	0.9314	0.9323	0.9305	0.9315
17.0	181	121	302	0.9329	0.9275	0.9307	0.9317	0.9286	0.9304
25.3	174	116	290	0.9285	0.9231	0.9263	0.9290	0.9246	0.9272
33.7	148	101	249	0.9249	0.9202	0.9229	0.9254	0.9197	0.9231
42.0	101	68	169	0.9213	0.9166	0.9193	0.9214	0.9144	0.9185
50.3	40	38	78	0.9190	0.9148	0.9168	0.9171	0.9088	0.9136
58.7	14	24	38	0.9219	0.8909	0.9028	0.9125	0.9029	0.9086
67.0	3	4	7	0.9055	0.8792	0.8913	0.9078	0.8969	0.9033

heights. β_0, β_1, β_2 for the bigtooth and trembling aspen equations were not significantly different ($P > 0.05$ in each case) using 2-sample t-tests.

Predicted *k*'s from the prediction equations for the 10 tree heights are shown in Table 3. Even though the predicted values from the big tooth aspen prediction equation are larger than those from the trembling aspen prediction equation, the differences in the predicted values at a given height vary from 0.0006 at 0.33+ ft. to 0.0109 at 67.0 ft. This indicates that the pooled prediction equation will yield adequate results for both species in most cases. The pooled predicted values are closer to the bigtooth predicted values because of the larger sample sizes for bigtooth aspen.

Fowler and Damschroder (1988) developed a bark factor prediction equation for red pine in Michigan of the form

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \frac{1}{X} + \hat{\beta}_2 \ln X$$

The data from their study were used to develop a prediction equation for red pine based on the equation form

Table 4. Prediction equations for bigtooth aspen, trembling aspen, aspen pooled, and red pine.

Prediction equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$S_{\hat{\beta}_0}$	$S_{\hat{\beta}_1}$	$S_{\hat{\beta}_2}$	R^2	n
Bigtooth aspen	0.9219	−0.000684	0.007556	0.000441	0.000037	0.000305	0.9890	10
Trembling aspen	0.9214	−0.000842	0.007588	0.001015	0.000079	0.000684	0.9491	10
Aspen pooled	0.9217	−0.000749	0.007564	0.000395	0.000032	0.000271	0.9911	10
Red pine	0.9167	−0.000575	0.022208	0.000275	0.000025	0.000202	0.9998	11

Note: All equations are of the form $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 \ln X$ where $\hat{Y}$ is estimated k and X is tree height in feet; n is the number of different heights used in developing the prediction equations. Stump height was 0.33+ and 0.5 ft for aspen and red pine, respectively.

Table 5. Average relative errors ($\overline{RE}$), minimum and maximum relative error values, and number of bigtooth aspen trees for each height class in feet (n) for the 3 stands (validation data sets). All *RE* values are percentages.

Au Sable (Stand 1)				Lake Superior (Stand 1)				Mackinaw (Stand 1)			
Ht. class	$\overline{RE}$	Min.–Max.	n	Ht. Class	$\overline{RE}$	Min.–Max.	n	Ht. class	$\overline{RE}$	Min.–Max.	n
0.33+	0.70	−1.26,2.43	4	0.33+	0.20	−3.78,6.51	16	0.33+	−0.13	−3.14,6.09	35
4.5	−0.62	−1.18,0.68	4	4.5	0.26	−1.55,6.08	16	4.5	−0.38	−2.59,2.01	35
8–9	−0.25	−1.60,1.87	4	8–9	0.30	−1.85,6.48	16	8–12	−1.20	−3.28,1.65	35
17–18	−0.17	−1.45,1.62	4	17–18	−0.27	−3.12,4.67	16	17–23	−1.55	−3.72,1.54	35
25–26	−0.22	−1.43,1.28	4	25–27	−0.59	−2.63,5.94	16	26–34	−1.91	−3.47,2.18	35
34–35	−0.05	−1.49,1.76	4	34–36	−0.91	−2.44,0.43	15	34–43	−1.96	−3.82,0.37	35
42–43	0.22	−2.37,2.63	3	43–45	−0.75	−2.90,2.08	15	43–53	−2.26	−4.14,−0.07	35
51–52	0.87	−0.69,2.43	2	51–54	0.70	−1.53,4.02	7	53–63	−2.29	−4.90,0.16	35
								63–70	−2.12	−5.18,0.99	28
								70–74	−1.14	−4.76,3.01	7

Table 6. Average relative errors ($\overline{RE}$), minimum and maximum relative error values, and number of trembling aspen trees for each height class in feet (n) for the 3 stands (validation data sets). All *RE* values are percentages.

	Au Sable (Stand 1)			Lake Superior (Stand 1)			Mackinaw (Stand 1)		
Ht. class	$\overline{RE}$	Min.–Max.	n	$\overline{RE}$	Min.–Max.	n	$\overline{RE}$	Min.–Max.	n
0.33+	1.66	−1.02,5.26	27	1.34	−6.56,8.01	38	0.71	−2.62,6.05	7
4.5	0.20	−1.96,5.59	27	−0.40	−2.97,3.01	38	−1.01	−2.59,0.50	7
8–12	0.77	−2.29,3.41	27	−0.40	−2.64,4.56	38	−0.69	−3.25,0.70	7
17–23	0.87	−1.94,5.43	27	−0.17	−2.92,5.67	38	−0.98	−4.28,0.66	7
26–34	1.49	−1.59,6.26	27	−0.33	−2.95,6.30	38	−0.82	−2.09,2.18	7
34–43	1.59	−1.46,7.05	26	0.69	−2.40,6.87	38	−0.57	−3.35,1.35	7
43–52	2.27	−1.96,5.74	18	0.99	−1.36,6.08	36	0.63	−1.79,3.00	7
52–60	3.83	−0.79,6.44	7	1.03	−2.04,6.18	22	−1.02	−0.78,0.58	2
60–70	0.04	—	1	2.06	−2.38,5.71	9	2.12	−2.97,7.21	2

used in the present study (Table 4). Skewness and kurtosis coefficients of the residuals for this equation were −0.37 and −0.60, respectively, indicating no serious departures from normality. This is also a very good prediction model for red pine in Michigan and could be used instead of the equation used by Fowler and Damschroder. Notice that the regression coefficients are considerably different than those of the 3 aspen prediction equations, indicating species differences. β_0, β_1, and β_2 for the pooled aspen equation and the red pine equations were significantly different ($P < 0.01$ in each case) using 2-sample t-tests.

The pooled aspen prediction model was validated on 3 independent data sets separately for each species (Tables 5 and 6) for various height classes in feet. Average relative errors as percentages ($\overline{RE}$) were calculated for each tree height class for each species using the formula

$$\overline{RE} = \sum_{i=1}^{n} RE_i/n$$

where

$$RE_i = \frac{\hat{Y}_i - Y_i}{Y_i}(100),$$

$\hat{Y}_i$ = predicted k for *ith* tree,
Y_i = observed k for the *ith* tree, and
n = number of trees with measurements in the specified height class.

For bigtooth aspen, the pooled prediction equation tends to underestimate k's for most tree heights (Table 5). For trembling aspen, the pooled prediction equation tends to somewhat overestimate k's for stump height and larger tree heights and underestimate k's for intermediate tree heights (Table 6).

Confidence intervals could be used to indirectly evaluate the accuracy of predictions. The validation results give a more direct evaluation of prediction accuracy. In evaluating the accuracy of the prediction equation, it must be remembered that sample sizes decrease greatly as tree height increases (Table 3). For discussions of weighted multiple linear regression, see Brownlee (1965), Draper and Smith (1981), and Steel and Torrie (1960).

COMMENTS

The pooled prediction equation appears to be adequate for estimating bigtooth and trembling aspen k for most situations. If more accurate estimates of k are desired, the separate prediction equations for bigtooth and trembling aspen should be used. The prediction equation given for red pine in this paper will, in all likelihood, perform as well as the equation given in Fowler and Damschroder (1988), as the predicted values for the equation given in this paper are closer to the actual values used to develop the equation, especially for large tree heights. Damschroder and Fowler (1988) discuss specific uses of bark factor prediction equations. □

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