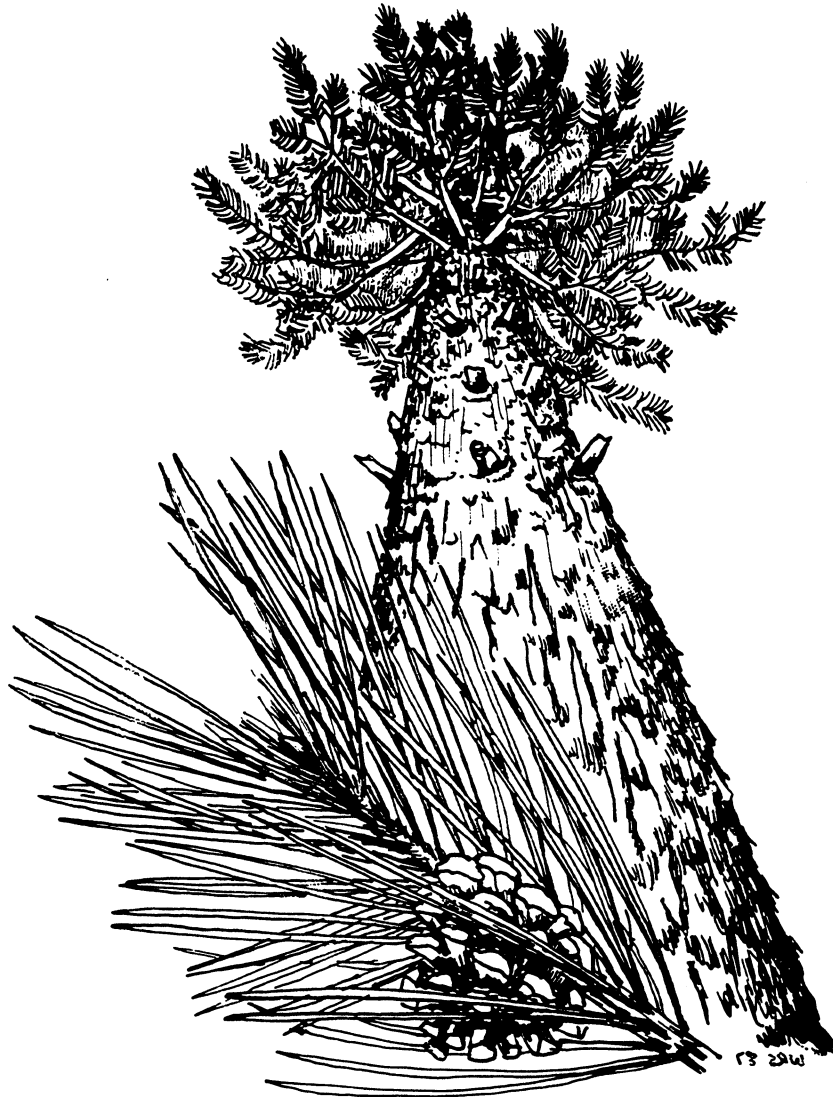


**ESTIMATION OF FOREST STAND PARAMETERS FOR ALL
TREES TO BE ROW-THINNED IN A RED PINE PLANTATION
TIMBER SALE IN MICHIGAN**



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Trees To Be Row-Thinned in a Red Pine
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Management Summary

Various cruising procedures were compared for estimating forest stand parameters, i.e., number of trees (*NOT*), basal area (*BA*), and volume (*V*) per acre, for all trees to be harvested in a row-thinned red pine plantation timber sale in Michigan where every third row is to be thinned. All procedures were compared on a one-acre study plot where all trees were enumerated.

The 100% tally procedure for all rows and every third row were compared with (1) sampling every K^{th} tree for all rows and every third row for various values of K starting with the first tree in the first row of the pertinent population, and (2) point sampling with 10 points using basal area factor (BAF) 10 and 20 prisms for north/south (N/S) and east-west (E/W) cruise lines. Monte Carlo simulation procedures were used to (1) compare taking samples based on every K^{th} tree for all possible K samples for various values of K for all rows and every third row, and (2) compare taking samples of 10 points using BAF 10 and 20 prisms for three systematic sampling procedures using N/S and E/W cruise lines and a simple random sampling procedure based on 100 trials of each procedure. The 100% tally procedure was considered a population enumeration, yielding population parameters for *NOT*, *BA*, and *V* per acre. The every K^{th} tree sample and point sample estimates were compared with these parameters. Variances and biases of the sample estimates were used in helping to decide which sampling procedures are best.

The study showed that the best procedure to use depends on what population you want to make inferences about: (1) the entire population of all rows as well as the population including every third row, or (2) only the population including every third row. The parameters for populations including every third row were surprisingly different, depending on whether you started with row 1, 2, or 3. This is due to the different tree mortality in the populations starting with rows 1, 2, or 3. This difference may not be as serious for large stands with much longer rows where things may be more even row-by-row. But this phenomenon should at least be considered in developing a sampling method.

For point sampling, do not use Carlson's Formula to estimate the number of cords/acre as it tends to underestimate red pine pulpwood volume. Cumulative VBARS yield results that are very close to individual VBARS and can be used with confidence that any bias is very small. Tentative

results indicate that it may be best to set cruise lines perpendicular to row direction, at least in terms of precision. A BAF 10 prism yields too many “count” trees. In most cases a BAF 20 prism should be used when sampling red pine plantations, yielding an average number of count trees between 6 and 10. For point sampling, the use of a sequential sampling procedure should be considered if it is important to obtain a certain level of precision.

We recommend the following sampling procedures for estimating NOT, BA, and V per acre for red pine plantation timber sales where every third row will be thinned:

1. Use point sampling when you need to estimate the parameters for the entire stand of all rows as well as for cutting every third row. For most stands, a sample of 20 to 30 points (i.e., approximately 160 to 240 trees) should yield adequate precision, depending on the variability of the stand. In any case, do not take a sample of more than 2% of the total number of trees in the stand. If NOT per acre is to be estimated, you must also measure the DBH of each count tree.
2. Use the every K^{th} tree procedure in only the rows to be cut when you only need to estimate the parameters for cutting every third row. For most stands, a sample of between 80 and 120 trees should yield adequate precision, depending on the variability of the stand. Never take any more than 2% of the total number of trees in all rows to be cut. For most stands, this will mean using K somewhere from 50 to 100. To estimate K determine the average distance between rows and between trees and estimate the number of trees per acre. You must measure the DBH and merchantable height of each sample tree to estimate BA and V per acre.

The recommended procedures will yield estimates of NOT, BA, and V per acre with adequate precision to meet timber sale requirements. For the every K^{th} tree procedure, count the number of trees in every third row so that the actual number of trees to be cut is known. If this is not feasible, then multiple K by the number of trees sampled to estimate the number of trees to be cut.

Whatever sampling procedure is used, more variable stands will require larger sample sizes for the same precision. The optimum sample size to use depends on the objectives of the study, desired precision, budget, time, and personnel limitations, cost, and ease of use in the field.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - TIMBER CRUISING PROCEDURES

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AUTHORS - Gary W. Fowler, Professor of Biometrics, School of Natural Resources and Environment, University of Michigan; and Nemah G. Hussain, Timber Sales Specialist, Forest Management Division, Michigan Department of Natural Resources.

Introduction

The Forest Management Division of the Michigan Department of Natural Resources (MDNR) uses a variety of cruising procedures to estimate forest stand parameters for all trees to be cut in row-thinned red pine plantation timber sales. The purpose of this study is to compare these procedures for a representative red pine timber sale and to recommend which procedure(s) to use.

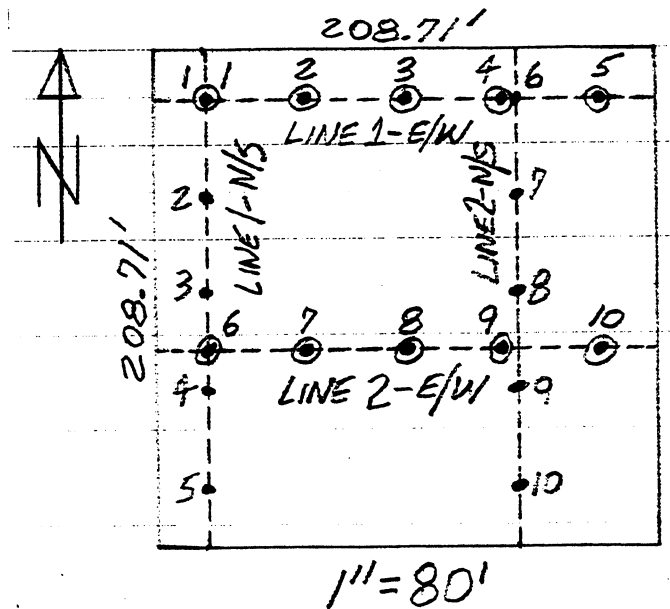


Figure 1. Boundary of the one-acre study plot. Dashed lines are the 4 cruise lines used for prism cruising. Lines 1 and 2 N/S and lines 1 and 2 E/W are 20.87 and 150.22 ft. and 20.87 and 125.22 ft. from the northwest corner of the study plot, respectively. Line 2-N/S was offset 25.00 ft. further from line 1-N/S because of a N/S road at about 125.22 ft. from line 1. Points 1 and 6 for both the N/S and E/W lines are 20.87 ft. from the west and north boundaries, respectively.

The Study Plot

A one-acre (208.71 x 208.71 ft.) study plot was established inside the Bachelor Block Timber Sale (52-002-96-01) in the E½, NW¼, and NE¼, SW¼, Sec. 17, T29N, R3W, Otsego County, Michigan in the Gaylord Forest Area (Figure 1). Boundary lines were established with a hand compass, fiberglass tape, and rangepoles. The study plot was located and all sampling and

measurement completed by the authors on August 7 and 8, 1996. The area of the sale was 42 acres, the place of the sale was the Gaylord Forest area of the MDNR, the proposal date was 10/31/95, and the permit will expire on 9/30/97.

Cutting in this sale will be a row-thinning where every third row will be removed. The estimated volume for the sale was 520 cords of pulpwood (i.e., 9 cords of aspen, 11 cords of white pine, 482 cords of red pine, and 18 cords of mixed hardwoods) and 14.50 MBF of red pine boltwood.

There were a total of 564 red pine pulpwood trees in 26 rows in the one-acre study plot. The average number of trees per row was 21.69, ranging from 14 to 27 trees. A pulpwood tree had to have a $DBH \geq 4.6$ in. and at least one 100-in. stick to a 3.6-in. minimum top. *DB?*

DBH was measured to the nearest one inch using a Biltmore stick and diameter tape. Merchantable height (MH) was measured as the number of 100-in. sticks to a 3.6-in. minimum top diameter using ocular estimation and a Merritt Hypsometer. Average DBH for all trees was 7.5 in., ranging from 5 to 10 inches. Average MH for all trees was 3.9 sticks, ranging from 1 to 6 sticks.

The X and Y coordinates of each tree were determined to the nearest foot by measuring the average E-W distance between each row and the distances between trees within a row using a fiberglass tape. The average distance from the west boundary of the plot to row 1 was determined as well as the distance from either the north boundary to the first tree in a row or the distance from the south boundary to the last tree in each row. For the one-acre plot, the rows were very close to being straight. Figure 2 shows a stem map of the 564 trees in the study plot.

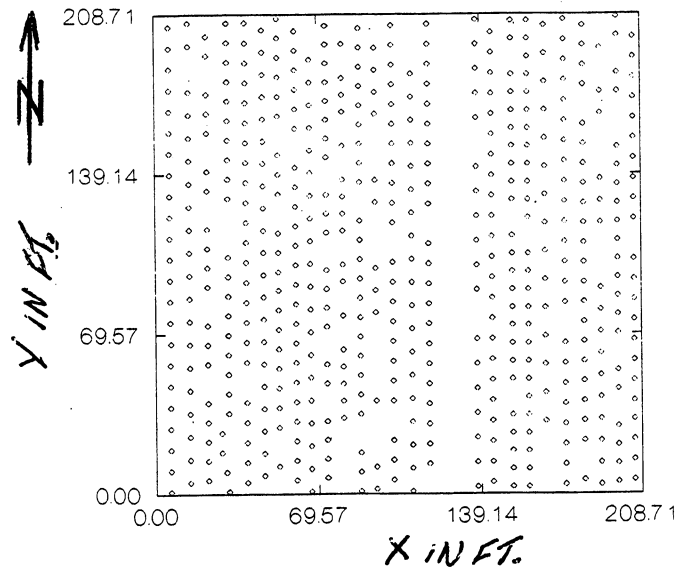


Figure 2. Stem map of the one-acre study plot. Notice the N/S road running through the plot and the missing trees in some of the 26 rows due to mortality.

Cruising Procedures Compared

The following cruising procedures were compared using the study plot:

- (1) One-third of 100% tally for all rows.
- (2) 100% tally of every third row starting with row 1, 2, or 3.
- (3) One-third of every K^{th} tree tally starting with the first tree for all rows ($K=50, 25, 10, 5, \text{ and } 2$).
- (4) Every K^{th} tree tally starting with the first tree for all trees in every third row starting with row 1, 2, or 3 ($K=50, 25, 10, 5, \text{ and } 2$).
- (5) Point sampling with 10 points using BAF 10 and 20 prisms. Two cruise lines of 5 points each were sampled (1) in a N/S direction, and (2) in an E-W direction (Figure 1).

NOTE: If the number of trees in the stand or the total number of rows sampled for the every K^{th} tree tally is not known, then K is used as the blowup factor to obtain per acre estimates. This yields biased estimates if the number of trees in the sale is not a multiple of K . In general, if $n < N/K$ or $n > N/K$, the bias is positive or negative, respectively, where n is the number of trees in the sample and N is the number of trees in the stand or the sale. Thus, if the first tree is included in the sample when $n \neq N/K$, this bias is positive and decreases as K decreases. In most timber sales, the number of trees in the stand or the number of trees to be cut is large and this bias is negligible.

For the 100% tally and every K^{th} tree methods, the following tree parameters were calculated:

- (1) Basal area (BA)= 0.005454 DBH^2 where DBH is in inches and BA is in sq. ft.
- (2) Volume in cords was calculated using the formula developed by Fowler and Hussain (1987a) where cubic-foot volume (CF) is

$$V_{CF} = 0.0700 \bullet \text{DBH}^{1.7349} \bullet \text{MH}^{0.848},$$

and cordwood volume (CW) is

$$V_{CW} = 0.96 \bullet V_{CF} / \text{CFPC}$$

where cubic-feet per cord (CFPC)=77, 79, 80, 81, 82, 83, and 84 for trees with MH=1, 2-3, 4, 5, 6, 7, and 8-10 sticks, respectively, and 0.96 is used to adjust for the 4-in. trim allowance associated with 100-in. sticks.

For the point sample method, the following per acre values were calculated for each point:

- (1) BA per acre = # count trees x BAF where BAF=10 or 20.

- (2) Volume in cords per acre for all count trees using Carlson's Formula:

$$V_C = \text{BAF} \frac{(\# \text{ count trees} \times \# \text{ sticks of all count trees})}{20}.$$

- (3) Volume in cords per acre (V_C) using the individual tree volume-basal area ratios (VBARs) for red pine pulpwood developed by Fowler and Hussain (1987b) where cubic-foot VBAR is 0.083, 0.171, 0.227, 0.271, 0.311, and 0.348 cu. ft./sq. ft. for MH= 1, 2, 3, 4, 5, and 6 sticks, respectively. Individual tree cordwood VBAR is

$$\text{VBAR}_C = 0.96 \text{VBAR}_{CF} / \text{CFPC}$$

where CFPC and the use of 0.96 is as described above.

For a given point,

$$V_C = \text{sum of the } \text{VBAR}_C \text{'s for all count trees} \times \text{BAF}$$

- (4) Volume in cords per acre (V_C) using the cumulative VBAR formula developed by Robert Ziel of the MDNR.

All of the above calculations, except for Carlson's Formula, are made for red pine in the MDNR-FMD timber program TSALE.

Results of Comparison

The population of red pine trees to be cut consists of cutting all trees in every third row. Table 1 shows the results of the 100% tally for all rows (i.e., all trees in the stand), and the 100% tally for every third row starting with row 1, 2, and 3. For purposes of comparing all cruising procedures, the 100% tally results yield population parameters for the 1-acre population.

Table 1. Number of trees (NOT), basal area in sq. ft. (BA), and volume in cords per acre for the 100% tally sampling procedures.

Inventory Procedure	NOT/acre	BA/acre	Vol./acre
1. 100% tally for all rows $\div 3$	188	58.54	16.63
2. 100% tally for every third row			
Starting with Row 1	202	63.57	17.98
Starting with Row 2	199	60.57	17.26
Starting with Row 3	163	51.49	14.64

For the 100% tally of all trees in the stand, the results shown in Table 1 are the parameters of the stand divided by 3. These values are used to estimate the parameters for all trees to be cut in every third row, assuming that which rows to be cut is not known until after sampling.

Table 2 summarizes the results of the “every K^{th} tree” cruising procedures starting with the first tree in the first row to be sampled. The number of trees (NOT) per acre is the actual number of trees which assumes that all trees will be counted in the appropriate population to be sampled. The values in parentheses are estimates of NOT per acre based on calculating $K \cdot n/3$ or $K \cdot n$ for sampling all rows or every third row, respectively, where n is the number of trees in the sample. These values are for the case where all trees are not counted in the appropriate population. Basal area (BA) per acre is in sq. ft., and pulpwood volume is in cords per acre. These estimates assume that all trees are counted for the appropriate population.

The results for point sampling are shown in Table 3, which includes BA/ac., and vol./ac. based on individual and cumulative VBARs and Carlson’s Formula.

Estimates from the “every K^{th} tree” (Table 2) and point sampling (Table 3) procedures will be compared with the appropriate population parameters (Table 1).

Table 2. Number of trees (NOT), basal area in sq. ft. (BA), and volume in cords per acre for the "every K^{th} " tree sampling procedures.

Inventory Procedure	NOT/acre	BA/acre	Vol./acre
1. Every K^{th} tree for all rows			
$K=50, n=12$	188 (200)	66.48	18.68
$K=25, n=23$	188 (191.7)	63.17	17.79
$K=10, n=57$	188 (190)	57.24	16.15
$K=5, n=113$	188 (188.3)	57.75	16.34
$K=2, n=282$	188 (188)	58.60	16.68
2. Every K^{th} tree for every 3rd row starting with Row 1			
$K=50, n=5$	202 (250)	64.78	18.42
$K=25, n=9$	202 (225)	58.88	16.73
$K=10, n=21$	202 (210)	59.39	16.84
$K=5, n=41$	202 (205)	62.07	17.58
$K=2, n=101$	202 (202)	62.57	17.73
3. Every K^{th} tree for every 3rd row starting with Row 2			
$K=50, n=4$	199 (200)	44.23	11.75
$K=25, n=8$	199 (200)	51.56	13.70
$K=10, n=20$	199 (200)	56.11	15.84
$K=5, n=40$	199 (200)	56.55	15.83
$K=2, n=100$	199 (200)	59.13	16.88
4. Every K^{th} tree for every 3rd row starting with Row 3			
$K=50, n=4$	163 (200)	55.34	16.68
$K=25, n=7$	163 (175)	52.20	15.46
$K=10, n=17$	163 (170)	53.24	15.34
$K=5, n=33$	163 (165)	52.86	15.08
$K=2, n=82$	163 (164)	53.01	15.05

Table 3. Basal area per acre (BA) in sq. ft., and volume per acre in cords for the N/S and E/W point samples based on BAF 10 and 20.

Inventory Procedure	BA/Acre	Vol./Acre		
		Individual VBARS	Cumulative VBARS	Carlson's Formula
1. Point Sampling - N/S				
BAF 10	54.33	14.59	14.34	13.45
BAF 20	62.00	16.67	16.40	15.40
2. Point Sampling - E/W				
BAF 10	52.67	14.26	14.00	13.17
BAF 20	56.00	15.13	14.86	13.97

The main results of the comparisons are:

- (1) One-third of the 100% tally for all trees in the population yields estimates for NOT/ac. that are 94.0, 94.5, and 115.3% of the 100% tallies for every third row starting with rows 1, 2, and 3, respectively. The respective percentages are 92.1, 96.6, and 113.7% for BA per acre, while they are 92.5, 96.5, and 113.6% for volume per acre.
- (2) The “every K^{th} tree” procedure for all trees yields sample sizes (n) of 12, 23, 57, 113, and 282 trees for $K=50, 25, 10, 5,$ and $2,$ respectively. Estimates of BA and volume considerably overestimated the respective parameters for $K=50$ and 25 . For $K=10$, estimates of BA and volume was 97.8 and 97.1% of the respective parameters. For $K=5$, estimates of BA and volume were 98.7 and 98.3% of the respective parameters. For $K=2$, estimates of BA and volume were 100.1 and 100.3% of the respective parameters.

If the number of trees in the population is not known (i.e., 188), estimates of NOT were 106.4, 102.0, 101.1, 100.2, and 100.0% of the true NOT for $K=50, 25, 10, 5,$ and $2,$ respectively.

- (3) Results for the “every K^{th} tree” procedure for every 3rd row depended on whether sampling started in row 1, 2, or 3.

Row 1 - Sample sizes were $n=5, 9, 21, 41,$ and 101 for $K=50, 25, 10, 5,$ and 2, respectively. Estimates of BA and volume somewhat overestimated and underestimated the respective parameters for $K=50$ and 25, respectively. For $K=10$, estimates of BA and volume were 93.4 and 93.7% of the respective parameters.

For $K=5$, estimates of BA and volume were 97.6 and 97.8% of the respective parameters. For $K=2$, estimates of BA and volume were 98.4 and 98.6% of the respective parameters.

If the number of trees in the population is not known (i.e., 202), estimates of NOT were 123.8, 111.4, 104.0, 101.5, and 100.0% of the true NOT for $K=50, 25, 10, 5,$ and 2, respectively.

Row 2 - Sample sizes were $n=4, 8, 20, 40,$ and 100 for $K=50, 25, 10, 5,$ and 2, respectively. Estimates of BA and volume considerably underestimated the respective parameters for $K=50$ and 25. For $K=10$, estimates of BA and volume were 92.6 and 91.8% of the respective parameters. For $K=5$, estimates of BA and volume were 93.4 and 91.7% of the respective parameters. For $K=2$, estimates of BA and volume were 97.6 and 97.8% of the respective parameters.

If the number of trees in the population is not known (i.e., 199), estimates of NOT were 100.5% of the true NOT for all K .

Row 3 - Sample sizes were 4, 7, 17, 33, and 82 for $K=50, 25, 10, 5,$ and 2, respectively. Estimates of BA and volume are considerably overestimated for $K=50$. For $K=25$, estimates of BA and volume were 101.4 and 105.6% of the respective parameters, while for $K=10$, estimates of BA and volume were 103.4 and 104.8% of the respective parameters. For $K=5$, estimates of BA and volume were 102.7 and 103.0% of the respective parameters, while for $K=2$, estimates of BA and volume were 103.0 and 102.8 of the respective parameters.

If the number of trees in the population is not known (i.e., 163), estimates of NOT were 122.7, 107.4, 104.3, 101.2, and 100.6% of the true NOT for $K=50, 25, 10, 5,$ and 2, respectively.

- (4) BAF 10 and 20 point samples were taken at 10 points (i.e., 5 points on each of 2 cruise lines) in (1) a N/S direction parallel to the tree rows and (2) a E/W direction perpendicular to the tree rows.

N/S - The BAF 10 and 20 prisms yielded estimates of BA/ac. that were 92.8 and 105.9% of the true BA. For the BAF 10 prism, estimates of vol./ac. using individual VBARs, cumulative VBARs, and Carlson's Formula were 87.7, 86.2, and 80.9%, respectively, of the true volume. The respective results for the BAF 20 prism were 100.2, 98.6, and 92.6%. One-hundred and sixty-three and 93 trees were sampled at the 10 points for the BAF 10 and 20 prisms, respectively.

The point sample estimates for BA and volume were reasonably precise. For BA, the AE%=5.2 and 5.6% for BAF=10 and 20, respectively. For BAF 10, volume estimates using individual VBARs, cumulative VBARs, and Carlson's Formula yielded AE%'s of 5.1, 5.1, and 5.1%, respectively. The respective AE%'s for BAF 20 were 6.5, 6.1, and 6.3%, respectively.

E/W - The BAF 10 and 20 prisms yielded estimates of BA/ac. that were 90.0 and 95.7% of the true BA. For the BAF 10 prism, estimates of vol./ac. using individual VBARS, cumulative VBARS, and Carlson's Formula were 85.7, 84.2, and 79.2%, respectively, of the true volume. The respective results for the BAF 20 prism were 91.0, 89.4, and 84.0% of the true volume. One-hundred and fifty-eight and 84 trees were sampled at the 10 points for the BAF 10 and 20 prisms, respectively.

The point sample estimates for BA and volume were reasonably precise, but not as precise as the point sample estimates for the N/S cruise lines. For BA, the AE% was 8.4 and 7.3% for BAF 10 and 20, respectively. For BAF 10, volume estimates using individual VBARS, cumulative VBARS, and Carlson's Formula yielded AE%'s of 8.5, 8.5, and 8.6%, respectively. The respective AE%'s for BAF 20 were 7.9, 7.8, and 8.0%, respectively.

Allowable error percent (AE%) is defined as

$$AE\% = \frac{2s_{\bar{x}}}{\bar{x}}(100),$$

where $s_{\bar{x}}$ is the sample standard error of the mean and \bar{x} is the sample mean calculated from the 10 points.

Simulation Analysis

A simulation analysis was made for both the "every K^{th} tree" and the point sample procedures to further examine the variability, precision, and accuracy of the estimates.

Every K^{th} tree sample

All possible samples were taken for the every K^{th} tree inventory procedure for all rows, and every 3rd row starting with row 1, 2, and 3, where $K = 50, 25, 20, 10, 5,$ and 2. The number of possible samples that could be randomly chosen from a given population of trees (i.e., 564 for all rows) is equal to K . Because the number of trees in the population $N = 564$ is, in general, not a multiple of K , sample sizes for the K samples are not all equal. However, the sample sizes will be within one tree of each other. The bias in estimates caused by this when using systematic sampling with a random start is negligible. For example, for $K = 50$, there are 14 samples of 12 trees and 36 samples of 11 trees out of $N = 564$ trees.

Table 4 shows the simulation results for the “every K^{th} tree” inventory procedure applied to the populations associated with all rows ($N = 564$ trees), and every third row starting with row 1 ($N = 202$ trees), row 2 ($N = 199$ trees), and row 3 ($N = 163$ trees) for various values of K . Note that the mean per acre values are not quite equal to the respective parameters found in Table 1 for many values of K . This is due to the small differences in sample size for a given K as discussed above. For $K = 2$ when sampling the all rows population, all sample sizes are equal since $564/2 = 282$. In this case, the mean per acre values are equal to the respective parameters. Sometimes even when not all sample sizes are equal, the mean per acre values are not different from the respective parameters, at least to 2 decimal places.

The variances per acre show that the precision of all estimates is very poor for $K = 50$ and poor for $K = 25$. In general, precision increases as K decreases.

Table 4. Mean and variance for number of trees (NOT), basal area (BA), and volume per acre for the "every K^{th} tree" inventory procedures based on the K possible samples for various value of K . The mean number of trees sampled is also given for each value of K .

Inventory Procedure	Mean Per Acre			Variance Per Acre			Mean # of Trees Sampled
	NOT	BA	Vol.	NOT	BA	Vol.	
1. Every K^{th} tree for all rows							
$K=50$	188	58.50	16.61	—	24.64	2.11	11.28
$K=25$	188	58.55	16.63	—	8.92	0.84	22.56
$K=10$	188	58.54	16.63	—	3.01	0.33	56.40
$K=5$	188	58.54	16.63	—	2.03	0.23	112.80
$K=2$	188	58.54	16.63	—	0.0037	0.0032	282.00
2. Every K^{th} tree for every 3rd row starting with Row 1							
$K=50$	202	63.59	17.99	—	79.34	6.93	4.04
$K=25$	202	63.63	18.00	—	20.56	2.01	8.08
$K=10$	202	63.58	17.99	—	8.72	0.60	20.20
$K=5$	202	63.57	17.98	—	4.23	0.38	40.40
$K=2$	202	63.57	17.98	—	1.01	0.0627	101.00
3. Every K^{th} tree for every 3rd row starting with Row 2							
$K=50$	199	60.56	17.26	—	63.17	5.72	3.98
$K=25$	199	60.58	17.26	—	30.02	3.14	7.96
$K=10$	199	60.60	17.27	—	14.83	1.52	19.90
$K=5$	199	60.59	17.27	—	8.41	1.05	39.80
$K=2$	199	60.57	17.26	—	2.09	0.14	99.50
4. Every K^{th} tree for every 3rd row starting with Row 3							
$K=50$	163	51.37	14.60	—	43.65	3.01	3.26
$K=25$	163	51.46	14.63	—	17.89	1.25	6.52
$K=10$	163	51.51	14.64	—	6.65	0.42	16.30
$K=5$	163	51.49	14.64	—	1.72	0.10	32.60
$K=2$	163	51.48	14.64	—	2.33	0.17	81.50

Population of all rows - For $K=10$, $AE\%=5.9$ and 6.9% for BA and volume, respectively. For $K=5$, the respective $AE\%$'s are 4.9 and 5.8% , and for $K=2$, the respective $AE\%$'s are 2.1 and 0.6% .

Populations for Every Third Row

Starting with 1st row - For $K=10$, $AE\%=9.3$ and 8.6% for BA and volume, respectively. For $K=5$ the respective $AE\%$ are 6.5 and 6.9% , and for $K=2$, the respective $AE\%$'s are 3.2 and 2.8% .

Starting with 2nd row - For $K=10$, $AE\%=12.7$ and 14.3% for BA and volume, respectively. For $K=5$, the respective $AE\%$'s are 9.6 and 11.9% , and for $K=2$, the respective $AE\%$'s are 4.8 and 4.3% .

Starting with 3rd row - For $K=10$, $AE\%=10.0$ and 8.9% for BA and volume, respectively. For $K=5$, the respective $AE\%$'s are 5.1 and 4.3% , and for $K=2$, the respective $AE\%$'s are 5.9 and 5.6% .

Note that the estimates for the population of all rows are used to estimate the respective parameters of the population of all rows. These estimates have already been divided by 3 so they can be used to estimate the parameters of the populations where every 3rd row has been thinned. Thus, the estimates based on sampling the population of all rows are biased in estimating the parameters of the populations where every 3rd row has been thinned. The bias will vary depending on whether thinning starts with row 1, 2, or 3 (See Table 1). For NOT, the true bias is -14 , -11 , and $+25$ for row 1, 2, and 3, respectively. For BA, the true bias is -5.03 , -2.03 , and 7.05 for row 1, 2, and 3, respectively. For volume, the true bias is -1.35 , -0.63 , and 1.99 for row 1, 2, and 3, respectively. Thus, the accuracy of these estimates in estimating the parameters of the populations where every 3rd row has been thinned is poor and varies depending on whether thinning starts with row 1, 2, or 3.

Point sample

One-hundred trials of BAF 10 and 20 points samples were taken at 10 points for 3 systematic sampling procedures and one random sampling procedure:

Systematic Sampling 1 - Two cruise lines of 5 points each were established.

- **N/S Lines** - The first line was selected randomly to start from 20.97 to 58.49 ft. from the western plot boundary, and the second line was selected to start 129.35 ft. east of the first line. The first point on each line was located 20.87 ft. from the northern plot boundary, and all other points were located on each line at an interval of 41.74 ft.
- **E/W Lines** - The first line was selected randomly to start from 20.87 to 83.49 ft. from the northern plot boundary, and the second line was selected to start 104.35 ft. south of the first line. The first point on each line was located 20.87 ft. from the western plot boundary, and all other points on each line at an interval of 41.74 ft.

This procedure was closest to the procedure used in the field to take the N/S and E/W BAF 10 and 20 samples. The second E/W line was offset 25.0 ft. further east due to the N/S road running through the plot (See Figures 1 and 2).

Systematic Sampling 2 - Two cruise lines of 5 points each were established as for Systematic Sampling 1 except that the second N/S line was selected to start 104.35 rather than 129.35 ft. east of the first line. In other words, this second line was not offset 25.0 ft. further east due to the N/S road running through the plot.

Systematic Sampling 3 - Two cruise lines of 5 points each were established randomly and points were systematically located along each line.

- **N/S Lines** - Each line was randomly selected to start from 20.87 to 187.84 ft. from the western plot boundary. The first point on each line was located 20.87 ft. from the northern plot boundary, and all other points were located on each line at an interval of 41.74 ft.
- **E/W Lines** - Each line was randomly selected to start from 20.87 to 187.84 ft. from the northern plot boundary. The first point on each line was located 20.87 ft. from the western plot boundary, and all other points were located on each line at an interval of 41.74 ft.

Random Sampling - Ten points were randomly located with their *X*- and *Y*- coordinates located randomly from 20.87 to 187.84 ft.

A computer program was written to take the 4 different types of samples of 10 points from the one-acre population. Simulation results are based on 100 trials (i.e., different samples) for each sampling procedure using a random number generator. The program estimated NOT, BA, and vol. per acre at each point using horizontal point sampling formulas (Avery and Burkhart 1994, Husch et al. 1982). The estimates calculated by the program are biased since no adjustment was made for statistical edge effect and there was a zone near the boundaries of the plot where no points were located (See Fowler and Arvanitis 1979, 1981; Arvanitis and Fowler 1987). The bias associated with statistical edge effect is negative while the bias associated with having no points near the boundaries depends on the size, density, and distribution of peripheral trees compared to interior trees. These biases can be alleviated by adjusting for statistical edge effect and taking points anywhere inside the plot, but this is not normally done by the MDNR when point sampling forest stands. The results of the 4 sampling procedures are comparable since all of them did not alleviate these biases.

The variances of the estimators were based on calculation of the variance of the 100 trials. You could also calculate the variance of the mean for each of the 100 samples of 10 points and calculate the mean of these 100 variances. However, these estimates of variances are biased for systematic sampling in general. These variances were also calculated but not used in this paper. For the E/W lines, these estimates were always considerably larger than the more accurate variances based on the 100 trials used in this paper. However, for the N/S lines, these estimates were somewhat to considerably smaller than the more accurate estimates except for one case for BAF 10 where the reverse was true. The difference between the 2 variances depends on the size, density, and distribution of trees as it affects variability within and between the samples of 10 points. It is dangerous to use the variance formula based on simple random sampling for systematic sampling, especially when the population of trees is not randomly located in the stand.

Table 5 shows the simulation results for the 4 sampling procedures to include the means and variances of the 100 trials for each sampling procedure. Remember that with point sampling we are estimating the population of all trees (i.e., all rows). Since we are interested in estimating the parameters for the rows that will be cut when every third row is cut, the values in Table 5 are based on estimates divided by 3.

The main results are as follows:

1. The bias of the estimates is larger for BAF 10 than BAF 20 for the N/S lines, with the reverse being true for the E/W lines. For the N/S lines, the bias decreases as you go from Systematic Sampling Procedure 1 to 2 to 3, with relatively little difference in bias among the 3 sampling procedures for the E/W lines.

Table 5. Mean and variance for NOT, BA, and volume per acre based on 100 randomly selected samples for the 4 point sampling procedures for BAF 10 and 20 and the N/S and E/W cruise lines.

Inventory Procedure	Mean Per Acre			Variance Per Acre		
	NOT	BA	Vol.	NOT	BA	Vol.
N/S Cruise Lines						
BAF 10						
Systematic Sampling 1	210.95	62.20	16.70	28.83	3.90	0.298
Systematic Sampling 2	199.36	59.49	15.99	431.52	18.17	1.266
Systematic Sampling 3	193.37	58.56	15.78	585.11	26.14	1.729
BAF 20						
Systematic Sampling 1	209.30	60.41	16.13	197.00	26.14	1.75
Systematic Sampling 2	198.85	58.72	15.72	724.77	33.89	2.55
Systematic Sampling 3	184.10	56.13	15.09	850.53	47.58	3.25
E/W Cruise Lines						
BAF 10						
Systematic Sampling 1	194.69	60.64	16.40	19.69	1.34	0.105
Systematic Sampling 2	196.29	60.69	16.40	17.27	1.02	0.082
Systematic Sampling 3	194.53	60.71	16.41	38.14	3.97	0.291
BAF 20						
Systematic Sampling 1	203.60	64.16	17.37	70.22	6.34	0.481
Systematic Sampling 2	204.46	64.15	17.36	66.31	5.46	0.413
Systematic Sampling 3	200.29	62.98	17.05	83.63	8.44	0.645
Random Sampling-BAF 10	188.55	58.56	15.80	230.81	11.27	0.764
Random Sampling-BAF 20	189.23	58.79	15.87	327.16	20.30	1.440

2. The precision of the estimates was considerably better for BAF 10 compared to BAF 20 for both N/S and E/W lines. This is because the average number of trees sampled per point for BAF 10 varied from 17.6 to 18.7 compared to 8.4 to 9.6 for BAF 20.
3. The precision of the estimates decreases as you go from Systematic Procedure 1 to 2 to 3 for the N/S lines. For the E/W lines, the precision was the same, other than sampling variation, for Procedures 1 and 2, with the precision being less for Procedure 3.

4. For the random sampling procedure, the estimates were less biased than systematic sampling procedures. The variances were larger than the systematic variances for the E/W lines. For the N/S lines, the variances were smaller than the systematic variances for Procedures 2 and 3 with mixed results for Procedure 1. The average number of trees sampled per point was 17.6 and 8.7 for BAF 10 and 20, respectively.
5. The allowable error percents (AE%'s) were higher for BAF 20 compared to BAF 10.

NOT - For the N/S lines, AE%=5.1, 20.8, and 25.0% for Systematic Sampling Procedures 1, 2, and 3, respectively, for BAF 10, with the respective AE%'s for BAF 20 being 13.4, 27.1, and 31.7%. For the E/W lines, AE%=4.6, 4.2, and 6.3%, respectively, for BAF 10, with the respective AE%'s=8.2, 8.0, and 9.1 for BAF 20. For the random sampling procedures, AE%=16.1 and 19.1% for BAF 10 and 20, respectively.

BA - For the N/S line, AE%=6.4, 14.3, and 17.5% for Systematic Sampling Procedures 1, 2, and 3, respectively, for BAF 10, with the respective AE%'s for BAF 20 being 16.9, 19.8, and 24%. For the E/W lines, AE%=3.8, 3.3, and 6.6%, respectively, for BAF 10, with the respective AE%'s=7.8, 7.3, and 9.2% for BAF 20. For the random sampling procedure, AE%=11.5 and 15.3% for BAF 10 and 20, respectively.

Vol. = For the N/S lines, AE%=6.5, 14.0, and 16.7% for Systematic Sampling Procedures 1, 2, and 3, respectively, for BAF 10, with the respective AE%'s for BAF 20 being 16.4, 20.3, and 23.9%. For the E/W lines, AE%=4.0, 3.5, and 6.6%, respectively, for BAF 10, with the respective AE%=8.0, 7.4, and 9.4% for BAF 20. For the random sampling procedure, AE%=11.1 and 15.1% for BAF 10 and 20, respectively.

In summary, precision is higher for BAF 10 compared to BAF 20 for the same number of points because about twice as many trees are sampled for BAF 10. Systematic Sampling Procedures 1 and 3 are most and least precise, respectively, for N/S lines. For E/W lines, Systematic Sampling Procedures 1 and 2 are most precise. The random sampling procedure was less precise than all 3 systematic sampling procedures for the E/W lines. For the N/S lines, the random sampling procedure was more precise than Systematic Sampling Procedures 2 and 3, and less precise than Systematic Sampling Procedure 1 except for BA and vol. using BAF 20.

Guidelines for Users

In selecting the best cruising procedures to use, it is important to know whether you need to estimate the parameters of the entire stand and for cutting every third row or only the parameters for cutting every third row. Also, remember that the results in this paper apply to one small one-acre population. Most stands in a timber sale will be considerably larger, usually ranging from 10 to 50 acres. The procedure used needs to combine ease of use in the field in a timely fashion and adequate precision in estimating NOT, BA, and volume per acre.

We recommend the following cruising procedures for red pine plantation timber sales where every third row will be thinned:

- (1) Use point sampling when you need to estimate parameters for the entire stand as well as for cutting every third row. Select a BAF that will yield about 6-10 count trees per point. For most plantations, BAF 20 should be used. Systematic sampling as presently used by the MDNR is appropriate for the size of stands related to timber sales. For most stands, assuming an average of 8 count trees per point, a sample of between 160 and 240 trees (i.e., 20 and 30 samples points, respectively) should yield adequate precision. Take fewer or more points if the variability of the stand is less or more than the average variability of

red pine plantations in the state. In any event, do not take a sample of more than 2% of the total number of trees in the stand. If you need to estimate NOT per acre, the DBH of each count tree must be measured. Be sure to check borderline trees when sampling.

Our example stand had 564 trees on one-acre. A 40-acre stand with this stem distribution would have 22,560 trees. Assume a BAF 20 yielded approximately 7 count trees per point (This could be estimated by taking several preliminary point counts with a BAF 20 prism). Twenty or 30 sample points would yield samples of approximately 140 or 210 trees, respectively. Never take any more than $2\% \times 22,560 = 451$ trees or 64 points.

- (2) Use the every K^{th} tree procedure in only the rows to be cut when you only need to estimate the parameters for cutting every third row. For most stands, a sample of between 80 and 120 trees should yield adequate precision. Take fewer or more trees if the variability of the stand is less or more than the average variability of red pine plantations in the state. In any event, do not take a sample of more than 2% of the total number of trees in the stand.

As calculated above, a 40-acre stand that had the same distribution of trees as our example one-acre stand would have 22,560 trees. One-third of this is 7,520 trees. For 80 or 120 trees in your sample, you would use $K=94$ or 63. Never take any more than $2\% \times 7,520 \approx 150$ trees, which yields a $K=50$.

To estimate K , determine the average distance between rows and between trees within a row and calculate your estimate for number of trees per acre. You could also count the number of trees in several rows to be cut, take an average, and multiply by the total number of rows to be cut. Then determine K as above. In taking your sample, count the number of trees in every third row so you know the actual number of trees that will be

cut. If this is not feasible, then use K to estimate the total number of trees to be cut as discussed earlier in this paper. You must measure the DBH and merchantable height of each sample tree in order to estimate BA and volume per acre. A tallywacker may be useful to keep a count of all trees in every third row.

- (3) For our study plot, Monte Carlo simulation with point sampling showed that using E/W cruise lines yielded estimates that were considerably more precise than estimates based on N/S cruise lines for the three systematic sampling procedures and both BAFs. Results were mixed when considering biases of the estimates and depended on the systematic sampling procedure, the forest parameter of interest, and the size of BAF used. Results for this study indicate that it may be best to use cruise lines perpendicular to the row direction in red pine plantations. This may be due to the systematic bias caused by the relationship of the cruise line to the plantation row being more serious for cruise lines parallel to the row direction. For our example stand, the N/S road may also be a factor. Further studies are needed examining the differences between cruise lines that are parallel and perpendicular to row direction.
- (4) For point sampling, do not use Carlson's Formula to estimate the number of cords per acre as it considerably underestimates. The cumulative VBARS and individual VBARS yield very similar results with the cumulative VBARS slightly underestimating the number of cords per acre.
- (5) If point sampling is used and a specified AE% (e.g., 10%) is desired, the possibility of using a sequential sampling rule based on some stopping rule should be considered. Determine the number of points as described above and take this number of points in the stand. Calculate the AE% of this sample. If the AE% of your sample is \leq the desired AE%, stop sampling. If not, take at least 5 more sample points (more if the AE% of your

original sample is considerably larger than the desired AE% -- e.g., 15% versus 10%) and calculate the AE% of your total number of points from the first and second stage of sampling. Repeat the sequential process until the AE% of your total number of points at that stage is \leq the desired AE%, then stop and calculate your estimates of NOT, BA, and volume per acre. Use volume data to make the AE% calculations. The use of an electronic data recorder would facilitate the use of this procedure in the field.

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