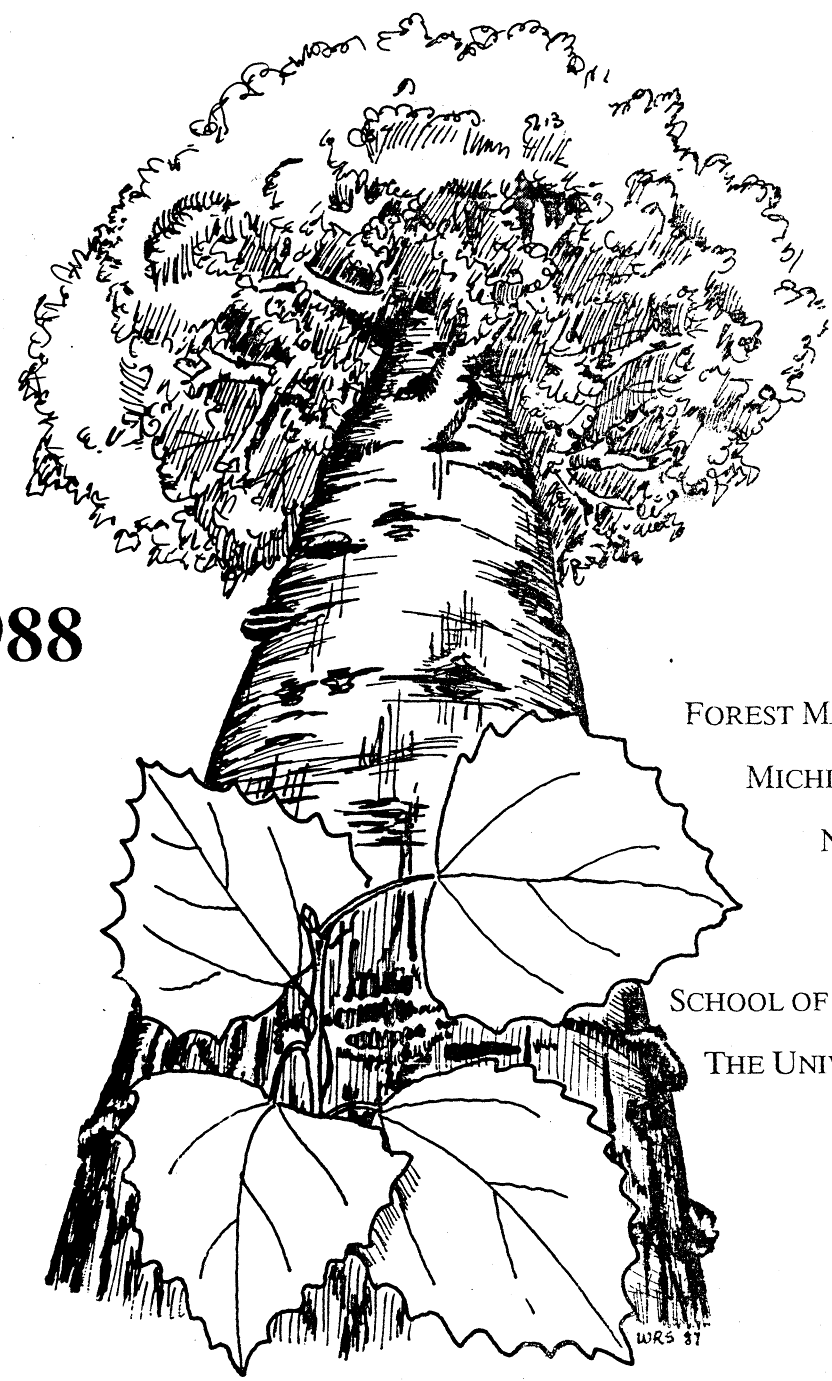
INDIVIDUAL TREE SAWTIMBER VOLUME EQUATIONS FOR ASPEN IN MICHIGAN

BY

Gary W. Fowler

Nemah G. Hussain



1988

FOREST MANAGEMENT DIVISION
MICHIGAN DEPARTMENT OF
NATURAL RESOURCES

SCHOOL OF NATURAL RESOURCES
THE UNIVERSITY OF MICHIGAN

Management Summary

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot individual tree sawtimber volume equations were developed for aspen in Michigan. Data used to develop these equations were collected from 24 aspen stands in Michigan (12 stands each from the Upper and Lower Peninsulas). Four stands each were sampled from each of the six state forests in Michigan.

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y \cdot x}$), and an independent validation data set for a series of linear and nonlinear regression equations indicated that nonlinear equations were most accurate for all types of volume. The new International 1/4-inch and Scribner board-foot volume equations yielded volume estimates (1) smaller than the values in Tables 1 and 2 of Gevorkiantz and Olsen (1955) for DBH's ≥ 18 in. with mixed results for smaller DBH's and (2) larger than the values in Tables 6 and 10 of Edminster et al. (1982) except for small merchantable heights. The new individual tree volume equations are:

1. Cubic-foot volume

$$\hat{V}_C = 0.3457 + 0.1126D^{1.513}H^{0.9467}$$

2. Doyle board-foot volume

$$\hat{V}_D = 1.311 + 0.04927D^{2.236}H^{1.101}$$

3. International 1/8-inch board-foot volume

$$\hat{V}_I = 3.208 + 0.4478D^{1.606}H^{1.077}$$

4. Scribner board-foot volume

$$\hat{V}_S = 2.484 + 0.3169D^{1.685}H^{1.076}$$

where D is DBH in inches and H is merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit. International 1/4-inch volumes are obtained by multiplying 0.905 times volumes obtained from Equation 3.

Multiple linear regression equations were developed to predict (1) one type of volume from another type and (2) Doyle, International 1/4-inch, and Scribner board-foot cubic-foot ratios as a function of D, H, and D and H.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

Michigan Department of Natural Resources
Forest Management Division

SUBJECT--INDIVIDUAL TREE VOLUME EQUATIONS

DATE -- 21 Sept. 88

TITLE -- Individual Tree Sawtimber Volume Equations for Aspen in Michigan

AUTHORS -- Gary W. Fowler, Professor of Biometrics, School of Natural Resources, University of Michigan, and Nemah G. Hussain, Timber Sales Specialist, Forest Management Division.

Background

Composite individual tree sawtimber board-foot (i.e., Scribner and International 1/4-inch rules) volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955). Volume equations and tables have been developed for aspen in Colorado (Edminster et al. 1982). The tables of Gevorkiantz and Olsen or some modifications of them are still widely used in Michigan for aspen.

Purpose

The purpose of this paper is to present new sawtimber cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot individual tree volume equations and tables for aspen in Michigan.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 1381 trees from 24 stands as follows:

1) 677 trees from 12 stands in the Upper Peninsula (i.e., 4 stands each in the Copper, Escanaba River, and Lake Superior state forests), and

2) 704 trees from 12 stands in the Lower Peninsula (i.e., 4 stands each in the Mackinaw, Au Sable, and Marquette state forests).

Measurements were taken on 728 bigtooth aspen and 653 trembling aspen trees. Stands were selected from the six forests to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May through August, 1986.

For the 24 stands, site index varied from 51 to 79, age varied from 47 to 70 years, basal area/acre varied from 70 to 186 sq. ft., average DBH varied from 7.7 to 11.9 in., average total height varied from 52.2 to 77.5 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 3.4 to 7.7 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b) developed pulpwood, sawtimber, and residual pulpwood individual tree cubic-foot volume equations from the total data set described above.

Sawtimber trees were defined as trees that had a least one 100-in. stick with a minimum inside bark top diameter no smaller than 7.6 inches. Sawtimber merchantable height is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches. There was a total of 946 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using

Smalian's formula, and summing the two volumes. For each 100-in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V = \frac{(B + b)L}{2}$ (Avery and Burkhart 1983)

Doyle: $V = 0.5D^2 - 4.0D + 8.0$ (Husch et al. 1982)

International 1/8-inch: $V = 0.44D^2 - 1.20D - 0.30$ (Husch et al. 1982)

Scribner: $V = 0.395D^2 - 0.99D - 2.15$ (Bruce and Schumacher 1950)

where

- V = volume in cubic feet or board feet,
- L = length of stick (100 in.) in ft.,
- B = cross-sectional area inside bark of large end of the stick in sq. ft.,
- b = cross-sectional area inside bark of small end of the stick in sq. ft., and
- D = diameter of small end of the stick inside bark in inches.

See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the three board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 7.6-in. top diameter limit. International 1/8-inch volumes were calculated for each tree, and all regression equations were developed using these volumes. International 1/4-inch volumes were obtained by multiplying 0.905 times the International 1/8-inch volumes.

Individual tree volume was regressed on various variables determined from tree DBH and merchantable height using multiple linear and nonlinear regression.

Results

The data set used to develop the regression equations consisted of 750 trees (? 80%) selected at random from the total of 946 trees. Fowler and Hussain (1987b) found no significant differences between pulpwood cubic-foot volume equations of the six forest areas and two species, so the data for both species and all stands were pooled before developing volume regression equations. All equations were based on the 750 trees with an average DBH = 10.8 in. (range: 8.0 to 16.7), average merchantable height = 2.8 sticks (range: 1 to 7), average cubic-foot volume = 12.20 (range: 2.82 to 54.49), average Doyle board-foot volume = 38.73 (range: 6.48 to 262.24), average International 1/8-inch board-foot volume = 72.83 (range: 15.99 to 380.35), and average Scribner board-foot volume = 62.34 (range: 13.14 to 335.85).

Cubic-foot and board-foot volume prediction equations

A comparison of various multiple linear regression and nonlinear regression equations based on goodness-of-fit and simplicity indicated that the following nonlinear prediction equation compared favorably to all other equations examined for cubic-foot and the three board-foot volumes:

$$\hat{V} = \hat{\beta}_0 + \hat{\beta}_1 D^{\hat{\beta}_2} H^{\hat{\beta}_3}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is merchantable height in 100-in. sticks to a 7.6-in. top diameter limit. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ are the sample regression coefficients related to the independent variables.

TABLE 1 shows the sawtimber volume prediction equations for cubic-foot and Doyle, International 1/8-inch, and Scribner board-foot volumes along with standard errors of the estimate ($s_{y \bullet x}$), and coefficients of determination (R^2) .

A cubic-foot volume table is shown in *TABLE 2*, and Doyle, International 1/4-inch, and Scribner board-foot volume tables are shown in *TABLES 3, 4*, and *5*, respectively. The International 1/4-inch board-foot values in *TABLE 4* were obtained by multiplying 0.905 times the values from Equation 3.

The values in our *TABLE 4* (International 1/4-inch board-foot volumes) are smaller for lower merchantable heights and larger for higher merchantable heights compared to the values in *TABLE 2* of Gevorkiantz and Olsen (1955) for DBH's less than 18 inches. The merchantable height where the values in our table become larger increases as DBH increases. For DBH's of 18 inches and over, our table, in general, gives smaller values.

TABLE 1. Estimated parameters ($\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of determination (R^2) for the cubic-foot and three board-foot volumes.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$s_{y \cdot x}$	R^2
(1) Cubic-foot[a]	0.3457	0.1126	1.513	0.9467	0.66	0.993
(2) Doyle[b]	1.311	0.04927	2.236	1.101	4.60	0.980
(3) International 1/8-inch[c]	3.208	0.4478	1.606	1.077	5.84	0.988
(4) Scribner[d]	2.484	0.3169	1.685	1.076	5.30	0.987

[a] $\hat{V} = 0.3457 + 0.1126D^{1.513}H^{0.9467}$

[b] $\hat{V} = 1.311 + 0.04927D^{2.236}H^{1.101}$

[c] $\hat{V} = 3.208 + 0.4478D^{1.606}H^{1.077}$

[d] $\hat{V} = 2.484 + 0.3169D^{1.685}H^{1.076}$

TABLE 2. Volume table showing cu. ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 1).

DBH	Merchantable Height in Sticks								
(inches)	1	2	3	4	5	6	7	8	9
9	3.5	6.4	9.2	12.0	14.7				
10	4.0	7.4	10.7	14.0	17.2	20.3			
11	4.6	8.5	12.3	16.1	19.8	23.4			
12	5.2	9.7	14.0	18.3	22.5	26.7	30.8		
13	5.8	10.9	15.8	20.6	25.4	30.1	34.7	39.4	
14	6.4	12.1	17.6	23.0	28.3	33.6	38.8	44.0	
15	7.1	13.4	19.5	25.5	31.4	37.3	43.1	48.8	54.5
16		14.7	21.5	28.1	34.6	41.1	47.4	53.8	60.1
17		16.1	23.5	30.7	37.9	45.0	52.0	58.9	65.8
18		17.5	25.6	33.5	41.3	49.0	56.6	64.2	71.8
19		19.0	27.7	36.3	44.8	53.1	61.4	69.7	77.8
20		20.5	29.9	39.2	48.4	57.4	66.4	75.3	84.1
21			32.2	42.2	52.0	61.8	71.4	81.0	90.5
22			34.5	45.2	55.8	66.2	76.6	86.9	97.1
23			36.9	48.4	59.7	70.8	81.9	92.9	103.8
24			39.3	51.6	63.6	75.5	87.3	99.1	110.7
25			41.8	54.8	67.6	80.3	92.9	105.3	117.7

TABLE 3. Volume table showing Doyle bd. ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 2).

DBH	Merchantable Height in Sticks								
(inches)	1	2	3	4	5	6	7	8	9
9	8	16	24	32	41				
10	10	20	30	40	51	62			
11	12	24	37	50	63	77			
12	14	29	44	60	76	93	110		
13	17	34	52	71	91	111	131	152	
14	19	40	62	84	107	131	155	179	
15	22	46	72	98	125	152	180	209	237
16		53	83	113	144	176	208	241	274
17		61	94	129	165	201	238	276	314
18		69	107	147	187	228	270	313	356
19		78	121	165	211	258	305	353	402
20		87	135	185	236	289	342	396	450
21			151	206	264	322	381	441	502
22			167	229	292	357	423	489	557
23			184	253	323	394	467	540	615
24			203	278	355	433	513	594	676
25			222	304	389	475	562	651	741

TABLE 4. Volume table showing International 1/4-inch bd. ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (0.905 x Equation 3).

DBH	Merchantable Height in Sticks								
(inches)	1	2	3	4	5	6	7	8	9
9	17	32	48	64	81				
10	19	37	56	76	95	116			
11	22	43	65	88	111	134			
12	25	49	74	100	127	154	181		
13	28	55	84	114	144	175	206	237	
14	31	62	95	128	162	196	231	267	
15	34	69	105	143	180	219	258	297	337
16		76	117	158	200	243	286	330	374
17		84	128	174	220	267	315	363	412
18		92	140	190	241	292	345	398	451
19		100	153	207	262	319	376	433	492
20		108	165	225	285	346	408	470	534
21			179	243	308	374	441	509	577
22			192	261	331	403	475	548	621
23			206	280	356	432	510	588	667
24			221	300	381	463	546	629	714
25			236	320	406	494	582	672	762

TABLE 5. Volume table showing Scribner bd. ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit (Equation 4).

DBH	Merchantable Height in Sticks								
(inches)	1	2	3	4	5	6	7	8	9
9	15	30	44	60	75				
10	18	35	53	71	89	108			
11	21	40	61	83	104	126			
12	23	46	71	95	120	146	172		
13	26	53	80	109	137	167	196	226	
14	30	60	91	123	155	188	222	256	
15	33	67	102	138	174	211	249	287	326
16		74	113	153	194	235	277	320	363
17		82	125	169	214	260	307	354	402
18		90	137	186	236	287	338	390	442
19		98	150	204	258	314	370	426	484
20		106	163	222	281	342	403	465	527
21			177	241	305	371	437	504	572
22			191	260	330	401	473	545	619
23			206	280	355	432	509	587	667
24			221	301	382	464	547	631	716
25			237	322	408	497	586	676	767

The values in our *TABLE 5* (Scribner board-foot volumes) are larger than those in *TABLE 1* of Gevorkiantz and Olsen for DBH's of 9 and 10 inches. The values in our table are smaller for lower merchantable heights and larger for higher merchantable heights for DBH's from 11 to 20 inches. The merchantable height where the values in our table become larger increases as DBH increases. For DBH's larger than 20 inches, our table, in general, gives smaller values. In comparing the two sets of tables, it should be kept in mind that our tables assume a 0.5-ft. stump, a minimum top diameter as close as possible to but no smaller than 7.6 inches, and log lengths of 8 ft., whereas the tables of Gevorkiantz and Olsen assume a 1.0-ft. stump, a minimum top diameter that is variable, is no smaller than 8.0 in., and can be considerably larger than 8.0 inches, and log lengths of 16 ft.

Our *TABLES 4* and *5* yield larger values than *TABLES 10* and *6*, respectively, of Edminster et al. (1982) except for small merchantable heights. It should be kept in mind that the tables of Edminster et al. assume a 1.0-ft. stump, a minimum top diameter of 5.6 inches, and log lengths of 16 and 8 ft.

Predicting One Type of Volume from Another

Multiple linear regression equations were developed to predict one type of volume from another using the 750 trees in the prediction data set. Equations were developed for predicting cubic-foot volume (CV) as a function of Doyle (DV), International 1/4-inch (IV), and Scribner (SV), DV as a function of CV, IV, and SV, IV as a function of CV, DV, and SV, and SV as a function of CV, DV, and IV. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in *TABLE 6*.

Predicting Board-foot Cubic-foot Ratios

Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios were calculated for each of the 750 trees in the prediction data set. Average board-foot cubic-foot ratios were 2.88 (range: 1.84 to 4.8), 5.21 (range: 3.85 to 6.32), and

TABLE 6. Regression equations for predicting cubic-foot (CV) and Doyle (DV), International 1/4-inch (IV), and Scribner (SV) board-foot volumes from the other three types of volumes.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CV} = 1.5015 + 0.3057\ DV - 0.000443\ DV^2$	0.993	0.66
$\hat{CV} = 0.5318 + 0.1853\ IV - 0.000082\ IV^2$	0.998	0.38
$\hat{CV} = 0.6617 + 0.1955\ SV - 0.000107\ SV^2$	0.998	0.40
$\hat{DV} = -2.2163 + 2.6741\ CV + 0.038958\ CV^2$	0.995	2.37
$\hat{DV} = -1.7112 + 0.5443\ IV + 0.000692\ IV^2$	0.997	1.65
$\hat{DV} = -1.5017 + 0.5835\ SV + 0.000645\ SV^2$	0.998	1.41
$\hat{IV} = -2.2678 + 5.2628\ CV + 0.018523\ CV^2$	0.998	2.48
$\hat{IV} = 4.7326 + 1.6969\ DV - 0.001770\ DV^2$	0.997	2.93
$\hat{IV} = 0.6249 + 1.0591\ SV - 0.000123\ SV^2$	1.000	0.43
$\hat{SV} = -2.5708 + 4.9298\ CV + 0.022232\ CV^2$	0.998	2.26
$\hat{SV} = 3.7357 + 1.6135\ DV - 0.001518\ DV^2$	0.998	2.16
$\hat{SV} = -0.5643 + 0.9430\ IV + 0.000114\ IV^2$	1.000	0.37

4.88 (range: 3.54 to 6.16) for Doyle, International 1/4-inch, and Scribner board-foot volumes, respectively.

Multiple linear regression equations were developed to predict the three types of board-foot cubic-foot ratios as a function of D, H, and D and H. These equations and their associated values of R^2 and $s_{y \cdot x}$ are shown in *TABLE 7.*

Board-foot cubic-foot ratios for the three types of board-foot volume as a function of H are shown in *TABLE 8*. Board-foot cubic-foot ratios for the three types of board-foot volume as a function of D are shown in *TABLE 9.*

Validation

The data set used to validate the prediction equations consisted of 196 trees, the remaining approximately 20% of the total of 946 trees. For each volume equation, the average relative error as a percent $(\overline{RE})$ was calculated where

$$\overline{RE} = \sum_{i=1}^{n} RE_i/n$$

and $RE_i = [(\hat{V}_i - V_i)/V_i] \times 100$, $\hat{V}_i$ and V_i are are the predicted and actual volumes for the i^{th} tree, and n is the number of trees in the validation data set (n = 196). The relative error as a percent for the sum of the predicted volumes was also calculated where

$$RE_S = \left[\left(\sum_{i=1}^{n} \hat{V}_i - \sum_{i=1}^{n} V_i \right) / \sum_{i=1}^{n} V_i \right] \times 100$$

and $\sum_{i=1}^{n} \hat{V}_i$ and $\sum_{i=1}^{n} V_i$ are the sum of the predicted and actual volumes, respectively.

TABLE 7. Regression equations for predicting Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as in function of D, H, and D and H.

Type of Ratio	Regression Equation	R^2	$s_{y \cdot x}$
Doyle	$\widehat{DCR} = -0.351 + 0.300D$	0.829	0.20
	$\widehat{DCR} = 2.006 + 0.310H$	0.787	0.23
	$\widehat{DCR} = 0.471 + 0.187D + 0.138H$	0.868	0.18
International 1/4-inch	$\widehat{ICR} = 3.269 + 0.180D$	0.501	0.30
	$\widehat{ICR} = 4.587 + 0.221H$	0.668	0.24
	$\widehat{ICR} = 4.587 - 0.000005D + 0.221H$	0.668	0.24
Scribner	$\widehat{SCR} = 2.562 + 0.215D$	0.597	0.26
	$\widehat{SCR} = 4.176 + 0.251H$	0.719	0.22
	$\widehat{SCR} = 3.827 + 0.043D + 0.212H$	0.725	0.22

TABLE 8. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of H.

H	Board-foot Cubic-foot Ratio		
(Sticks)	DCR	ICR	SCR
1	2.3	4.8	4.4
2	2.6	5.0	4.7
3	2.9	5.2	4.9
4	3.2	5.5	5.2
5	3.6	5.7	5.4
6	3.9	5.9	5.7
7	4.2	6.1	5.9
8	4.5	6.4	6.2
9	4.8	6.6	6.4
10	5.1	6.8	6.7

TABLE 9. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D.

D	Board-foot Cubic-foot Ratio		
(Inches)	DCR	ICR	SCR
9	2.3	4.9	4.5
10	2.6	5.1	4.7
11	2.9	5.2	4.9
12	3.2	5.4	5.1
13	3.5	5.6	5.4
14	3.8	5.8	5.6
15	4.1	6.0	5.8
16	4.4	6.1	6.0
17	4.7	6.3	6.2
18	5.0	6.5	6.4
19	5.3	6.7	6.7
20	5.6	6.9	6.9
21	5.9	7.0	7.1
22	6.2	7.2	7.3
23	6.5	7.4	7.5
24	6.8	7.6	7.7
25	7.1	7.8	7.9

For the validation data set, average DBH = 10.8 in. (range: 8.4 to 15.3), average merchantable height = 2.8 sticks (range: 1 to 6), average cubic-foot volume = 12.20 (range: 2.78 to 37.53), average Doyle board-foot volume = 38.90(6.48 to 155.4), average International 1/8-inch. board-foot volume = 72.98 (range: 15.99 to 246.17), and average Scribner board-foot volume = 62.48 (range: 13.14 to 215.48).

TABLE 10 shows $\overline{RE}$, range of RE_i, and RE_s for the cubic-foot and Doyle, International 1/8-inch, and Scribner board-foot prediction equations.

TABLE 10. Average relative error ($\overline{RE}$), range of RE_i, and relative error for the sum of the predicted values (RE_s) for the four volume prediction equations (equations 1-4).

Prediction Equation	$\overline{RE}$	Range of RE_i	RE_s
(1) Cubic-foot	0.9	-14.8 to 21.2	0.23
(2) Doyle	1.7	-21.5 to 24.6	-0.15
(3) International 1/8-inch	1.1	-16.6 to 16.5	0.13
(4) Scribner	1.1	-17.3 to 17.5	-0.013

$\overline{RE}$ was less than 2% and $|RE_s|$ was less than 0.25% for all volume prediction equations. Plots of RE_i versus V_i showed no prediction problems for the range of V_i's in the validation data set except for small values of V_i. The volume prediction equations over-predict volume for cubic-foot volumes less than 4 and Doyle, International 1/4-inch and Scribner board-foot volumes less than 15, 20, and 15, respectively.

Guidelines for Users

We recommend the use of Equations 1-4 for estimating individual tree volume for aspen in Michigan for most cruising situations. The merchantability limits assumed by these equations closely approximate merchantability limits used by the DNR.

If the user wants to predict one type of volume from another, the appropriate equation in *TABLE 6* can be used. If the user needs to estimate board-foot cubic foot ratios for one of the types of board-foot volumes, the appropriate equation in *TABLE 7* can be used.

Literature Cited

Avery, T. E. and H. E. Burkhart. 1983. Forest Measurements. McGraw-Hill Book Co. N.Y. 331 p.

Bruce, D. and F. Schumacher. 1950. Forest Mensuration. 3rd Ed. McGraw-Hill Book Co. N.Y. 483 p.

Edminster, C. B., H. T. Mowrer, and T. E. Hinds. 1982. Volume tables and point-sampling factors for aspen in Colorado. USDA Forest Service Research Paper RM-232. Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado 16 p.

Fowler, G. W. and N. G. Hussain. 1987a. Bark factor equation for aspen. Michigan DNR Forestry Information Leaflet 2-87. 2p.

Fowler, G. W. and N. G. Hussain, 1987b. Individual tree volume equations for aspen in Michigan. Michigan Department of Natural Resources. 22 p.

Gevorkiantz, S. R. and L. P. Olsen. 1955. Composite volume tables of timber and their application in the Lake States. USDA Technical Bulletin 1104. 51 p.

Husch, B., C. I. Miller, and T. W. Beers. 1982. Forest Mensuration. John Wiley and Sons, Inc. N.Y. 402 p.

MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES
FOREST MANAGEMENT DIVISION