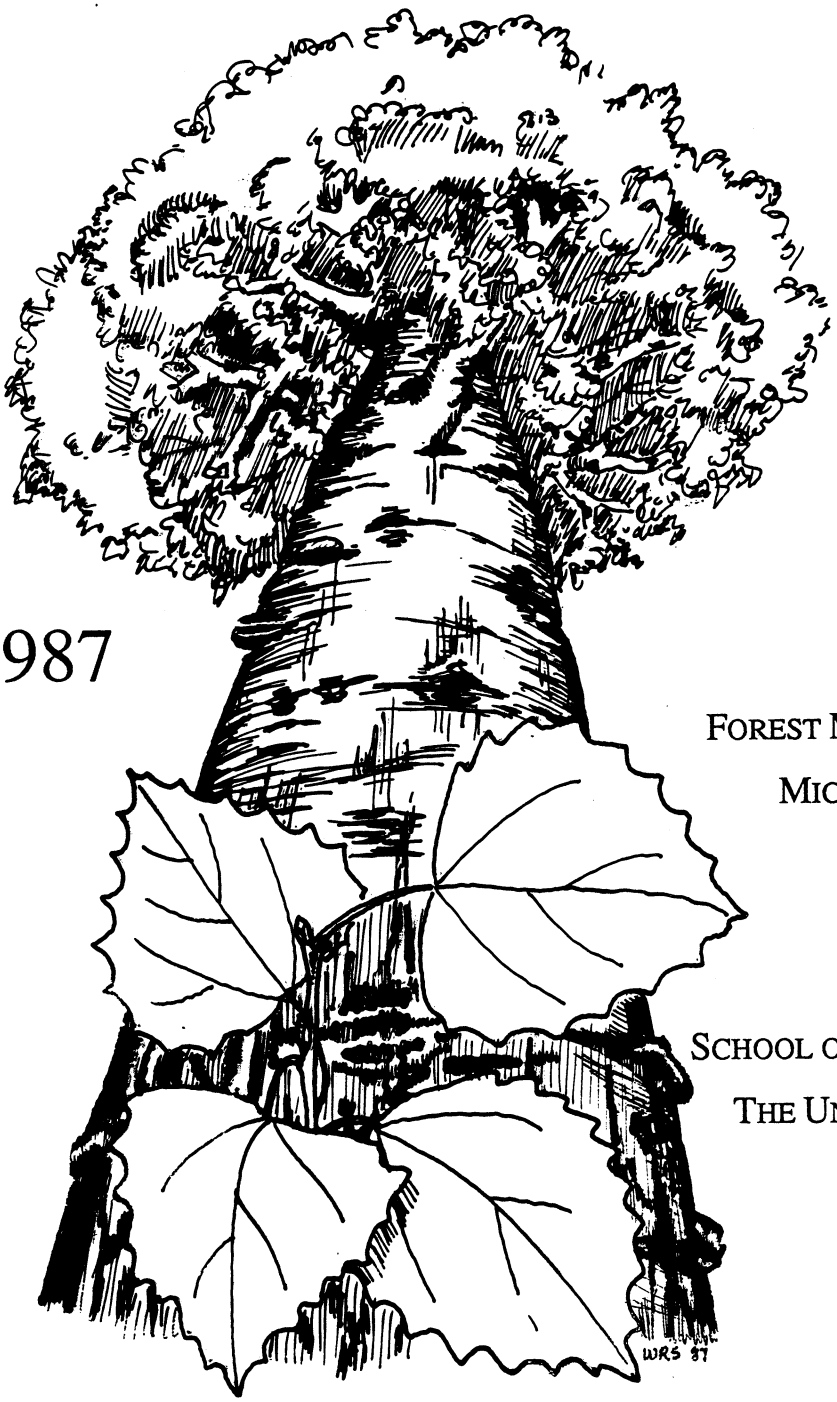
VOLUME-BASAL AREA RATIO EQUATIONS FOR ASPEN IN MICHIGAN

BY

Gary W. Fowler

Nemah G. Hussain



1987

FOREST MANAGEMENT DIVISION
MICHIGAN DEPARTMENT OF
NATURAL RESOURCES

SCHOOL OF NATURAL RESOURCES
THE UNIVERSITY OF MICHIGAN

Volume-Basal Area Ratio Equations for Aspen in Michigan

by

Gary W. Fowler and Nemah G. Hussain

1987

Forest Management Division
Michigan Department of Natural Resources
and
School of Natural Resources
The University of Michigan

Management Summary

New pulpwood, sawtimber, and residual pulpwood cubic-foot volume-basal area ratio (VBAR in cu.ft./sq.ft.) equations were developed for aspen in Michigan. Data used to develop these equations were collected from 24 aspen stands in Michigan (12 stands each from the Upper and Lower Peninsulas). Four stands were sampled from each of the 6 state forests in Michigan.

VBAR equations using diameter at breast height (DBH) and height independent variables yielded somewhat higher coefficients of determination (R^2), lower standard errors of estimate ($s_{y \cdot x}$), and more accuracy than VBAR equations using height independent variables. However, the differences between the 2 sets of equations are very small, indicating that the use of the simpler height VBAR equations is justified for almost all cruising situations. The new pulpwood VBAR equations based on height independent variables will usually yield volume estimates from 3 to 7% higher than estimates based on the VBARs presently used by the Michigan Department of Natural Resources.

We recommend the use of the following VBAR equations in most cruising situations for aspen:

1. Pulpwood VBAR

$$\hat{VBAR}_P = 9.1492+3.3168 \cdot PH-6.7480 \cdot \frac{1}{PH}$$

2. Sawtimber VBAR

$$\hat{VBAR}_S = 7.6691+4.1810 \cdot SH-4.3016 \cdot \frac{1}{SH}$$

3. Residual pulpwood VBAR

$$\hat{VBAR}_{RP} = \hat{P} \cdot \hat{VBAR}_P, \text{ where}$$

$$\hat{P} = -0.2752+0.9937 \cdot \frac{RH}{PH}+0.1692 \cdot \frac{1}{RH}+0.02340 \cdot \frac{PH}{SH}$$

In the above equations, PH is pulpwood merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit, SH is sawtimber merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit, RH is the

residual number of pulpwood sticks above and beyond sawtimber sticks, and $\hat{P}$ is the predicted proportion of residual pulpwood VBAR. For trees with both sawtimber and residual pulpwood volume, we recommend that sawtimber volume be determined using $\hat{VBAR}_S = \hat{VBAR}_P - \hat{VBAR}_{RP}$. For trees with just sawtimber, Equation 2 above should be used. Pulpwood and residual pulpwood rough cord $\hat{VBAR}$s can be determined from the respective cubic foot $\hat{VBAR}$s using appropriate cu.ft./cd. conversion values.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - VOLUME-BASAL AREA RATIO EQUATIONS

DATE - 30 September 87

TITLE - Volume-Basal Area Ratio Equations for Aspen in Michigan.

AUTHORS - Gary W. Fowler, Professor of Biometrics, School of Natural Resources, University of Michigan, and Nemah G. Hussain, Timber Sales Specialist, Forest Management Division, Michigan Department of Natural Resources.

Background

The Michigan Department of Natural Resources (DNR) developed volume-basal area ratios (VBARs) expressed in volume units per sq. ft. in 1974 to be used in estimating pulpwood volume (cords) and sawtimber volume (board feet, International-$\frac{1}{4}$ Rule) based on prism cruising. Three tally sheets were developed based on these VBARs (R 4137, 4144, and 4145). Because of perceived inaccuracies in the pulpwood VBARs and the time involved in using these tally sheets, the DNR is presently using Carlson's Formula for some pulpwood volume prism cruising. The formula for the Carlson Method when using a basal area factor of 10 is

$$\text{vol. in cds./acre} = \frac{\text{Total no. of trees} + \text{Total no. of sticks}}{2}$$

Purpose

The purpose of this paper is to present new pulpwood, sawtimber, and residual pulpwood cubic-foot VBAR equations and tables for aspen in Michigan. Rough cord VBAR equations and tables are also given for pulpwood and residual pulpwood.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 1381 trees from 24 stands as follows:

1) 570 trees from 12 stands in the Upper Peninsula (i.e., 4 stands each in the Copper, Escanaba River, and Lake Superior state forests), and
2) 591 trees from 12 stands in the Lower Peninsula (i.e., 4 stands each in the Mackinaw, Au Sable, and Marquette state forests).

Measurements were taken on 728 bigtooth aspen and 653 trembling aspen trees. Stands were selected from the six forests to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1986.

For the 24 stands, site index varied from 51 to 79, age varied from 47 to 70 years, basal area/acre varied from 70 to 186 sq.ft., average DBH varied from 7.7 to 11.9 in., average total height varied from 52.2 to 77.5 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 3.4 to 7.7 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5'), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987).

Merchantable height to an approximate 3.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum inside bark top diameter no smaller than 3.6 inches. This minimum top diameter was decreased for trees where the last stick had a minimum top diameter of 3.6 in. at a length of at least 6 ft. and a full 100-in. stick could be cut

from the tree. For felled trees, the last stick sometimes had a minimum top diameter less than 3.6 inches. Merchantable height to an approximate 7.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. Pulpwood and sawtimber volumes were determined by summing up the volumes of sticks to approximate 3.6-in. and 7.6-in. top diameter limits, respectively. Residual pulpwood volume above and beyond sawtimber volume was determined as the difference between pulpwood and sawtimber volumes. The pulpwood, sawtimber, and residual pulpwood VBARs were obtained for each tree by dividing the appropriate cubic-foot volume of the tree by the basal area in sq.ft. of the tree at 4.5 ft. from the ground. Pulpwood VBARs in rough cords were obtained by dividing cubic-foot VBARs by 73-80 cu.ft./cd., depending on the average DOB of all sticks for trees with merchantable heights varying from 1-9 sticks (Taras 1956, Avery and Burkhart 1983), and multiplying the result by 0.96 to compensate for the extra 4 in. of stick length beyond 8 ft. Residual pulpwood VBARs in rough cords were obtained by dividing cubic-foot VBARs by 75 cu.ft./cd. and multiplying by 0.96.

VBAR was regressed on various forms of tree height and DBH using multiple linear regression.

Results

The data set used to develop the regression equations consisted of approximately 80% of the trees in each of the 24 stands, yielding a total of 1161 trees. Individual tree pulpwood volume equations of the form $V=\beta_o+\beta_1 D^2H$ were

developed separately for trembling (544 trees) and bigtooth (617 trees) aspen for each state forest, where V is cubic-foot volume, D is DBH, and H is pulpwood merchantable height. There were no significant differences between the equations of the 2 aspen species using the paired comparison t-test for intercepts (β_o) and regression coefficients (β_1) with a level of significance α=0.05. Therefore, the data for both species were pooled before developing VBAR regression equations.

The pulpwood VBAR equations were based on 1161 trees with average DBH = 9.7 in. (range: 4.6 to 16.7), average merchantable pulpwood height = 5.5 sticks (range: 1 to 9), average total height = 64.8 ft. (range: 34.3 to 95.8), and average cubic foot pulpwood volume = 14.80 cu.ft. (range: 0.81 to 58.46). A pulpwood tree had to have a DBH $\geq$4.6" with at least one 100-in. stick having a minimum top diameter $\geq$3.6 inches.

The sawtimber VBAR equations were based on 802 trees with an average DBH = 10.8" (range: 8.0 to 16.7), average merchantable sawtimber height = 2.8 sticks (range: 1 to 7), and average cubic-foot sawtimber volume = 12.20 (range: 2.78 to 54.49). A sawtimber tree had to have at least one 100-in. stick with a minimum top diameter $\geq$7.6 inches.

The residual VBAR equations were based on the same 802 trees used to develop the sawtimber VBAR equations. The number of residual pulpwood sticks varied from 2 to 6, 2 to 6, 1 to 5, 1 to 5, 1 to 4, and 1-3 for 1, 2, 3, 4, 5, and 6 sawtimber sticks, respectively. Two trees with 7 sawtimber sticks had 2 residual pulpwood sticks.

Pulpwood and sawtimber VBAR prediction equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated that the following prediction equations

compared favorably to all other equations examined for pulpwood and sawtimber VBARs:

1. Height independent variables

   $\hat{VBAR} = \hat{\beta}_o + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H}$, and

2. Height and DBH independent variables

   $\hat{VBAR} = \hat{\beta}_o + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H} + \hat{\beta}_3 D + \hat{\beta}_4 \frac{1}{D}$,

where $\hat{VBAR}$ is predicted VBAR, H is merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit (PH), or merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit (SH) for pulpwood and sawtimber VBAR, respectively, and D is DBH in inches. $\hat{\beta}_o$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ are the sample regression coefficients related to the various independent variables. There were no significant differences between the state forests in the U.P. and state forests in the L.P using the one-way analysis of variance for intercepts (β_o) and the two regression coefficients (β_1 and β_2) with the height only models for pulpwood or sawtimber VBAR prediction equations using a level of significance α=0.05. Therefore, all of the data from the 24 stands were pooled for each type of VBAR equation to develop one pooled prediction equation for Michigan.

Table 1 shows the pulpwood and sawtimber VBAR prediction equations along with the sample sizes (n), standard errors of estimate ($s_{y \cdot x}$), and coefficients of determination (R^2) for the height only models. Pulpwood and sawtimber cubic-foot VBAR values for various heights are shown in Table 2. Pulpwood rough cord VBAR values are shown in Table 3. Table 3 also shows the pulpwood rough cord VBARs presently used by the DNR. The new VBARs are -12.8, 10.4, 7.5, 4.4, 2.7, 3.3, 3.8, and 6.9% larger than VBARs in tables presently used by the DNR for trees with 1, 2, 3, 4, 5, 6, 7, and 8 sticks, respectively. These values were obtained by dividing the pulpwood VBAR values in Table 2 by 73, 75, 76, 77,

Table 1. Estimated intercepts ($\hat{\beta}_o$), regression coefficents $\hat{\beta}_1$ and $\hat{\beta}_2$, sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of multiple determination (R^2) for the pulpwood and sawtimber cubic-foot VBAR prediction equations with independent variables based on height only.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	n	$s_{y \cdot x}$	R^2
(1) Pulpwood[a]	9.1492	3.3168	-6.7480	1161	1.84	0.897
(2) Sawtimber[b]	7.6691	4.1810	-4.3016	802	1.06	0.979

[a] $\hat{VBAR} = 9.1492+3.3168 \cdot PH-6.7480 \cdot (1/PH)$.

[b] $\hat{VBAR} = 7.6691+4.1810 \cdot SH-4.3016 \cdot (1/SH)$.

Table 2. Pulpwood and sawtimber VBARs in cu.ft./sq.ft. for various values of merchantable height.

Height (sticks)	VBAR		Height (sticks)	VBAR	
	Pulpwood	Sawtimber		Pulpwood	Sawtimber
1	5.72	7.55	6	27.93	32.04
2	12.41	13.88	7	31.40	36.32
3	16.85	18.78	8	34.84	40.58
4	20.73	23.32	9	38.25	44.82
5	24.38	27.71	10	41.64	49.05

Table 3. Rough cord VBARs for various numbers of pulpwood sticks from the new pulpwood VBAR prediction equation and from DNR tally sheets R 4137 and 4144.

Height (sticks)	VBAR		Height (sticks)	VBAR	
	New	DNR		New	DNR
1	0.075	0.086	6	0.344	0.333
2	0.159	0.144	7	0.382	0.368
3	0.216	0.201	8	0.418	0.391
4	0.262	0.251	9	0.459	--
5	0.304	0.296	10	0.500	--

78, 79, and 80 cu.ft./cd. for trees with 1, 2-3, 4, 5, 6, 7, and 8-10 sticks, respectively, and multiplying the result by 0.96.

Table 4 shows the pulpwood and sawtimber VBAR prediction equations for the height and diameter models. Note that R^2 and $s_{y \cdot x}$ for these equations are

Table 4. Estimated intercepts ($\hat{\beta}_o$), regression coefficents $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$, sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of multiple determination (R^2) for the pulpwood and sawtimber cubic-foot VBAR prediction equations with independent variables based on height and diameter.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	n	$s_{y \cdot x}$	R^2
(3) Pulpwood[a]	10.4787	3.7000	-8.4412	-0.3788	5.3491	1161	1.73	0.908
(4) Sawtimber[b]	27.9584	5.1000	-3.3111	-1.4695	-79.6189	802	0.82	0.987

[a] $\hat{VBAR} = 10.4787+3.7000 \cdot PH-8.4412 \cdot (1/PH)-0.3788 \cdot D+ 5.3491 \cdot (1/D)$.

[b] $\hat{VBAR} = 27.9584+5.1000 \cdot SH-3.3111 \cdot (1/SH)-1.4695 \cdot D-79.6189 \cdot (1/D)$.

larger and smaller, respectively, than for the respective equations based on height only (Table 1). Pulpwood and sawtimber cubic-foot VBAR tables based on these equations are shown in Tables 5 and 6, respectively, in the Appendix. A pulpwood rough cord VBAR table is given in Table 7 in the Appendix. The values in this table were obtained in the same way Table 3 values were obtained from Table 2 values for the height only model for pulpwood VBAR. The same cubic-feet per rough cord values for merchantable heights of 1 to 10 sticks were used to obtain Table 7 as were used to obtain Table 3.

Residual pulpwood VBAR prediction equations

Residual pulpwood VBAR in a sawtimber tree was estimated using two methods: 1) Use of a prediction equation for residual VBAR, and 2) multiplication of the estimated proportion of residual pulpwood VBAR in a sawtimber tree by the estimated pulpwood volume in the tree.

Method 1 -- Two prediction equations were developed for residual pulpwood cubic-foot VBAR:

1. Height independent variables

(5) $$\hat{VBAR}=-11.4459+17.0187\cdot\frac{RH}{PH}+3.0053\cdot RH+8.5680\cdot\frac{1}{RH}$$

$R^2=0.930$, $s_{y\cdot x}=1.33$, n=802

2. Height and diameter independent variables

(6) $$\hat{VBAR}=-53.7901+4.6106\cdot\frac{RH}{PH}+3.0807\cdot RH+5.0121\cdot\frac{1}{RH}$$
$$+1.5863\cdot D+350.3108\cdot\frac{1}{D}$$

$R^2=0.966$, $s_{y\cdot x}=0.93$, n=802

where $\hat{VBAR}$ is predicted residual pulpwood VBAR and RH is the residual number of pulpwood sticks above and beyond sawtimber sticks.

Table 8 in the Appendix shows residual cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 5. Table 9 in the Appendix shows residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks. VBARs from Table 8 were divided by 75 cu.ft./cd and then multiplied by 0.96 to obtain Table 9 values. A residual cubic-foot or rough cord VBAR table can also be developed from Equation 6 for various values of SH, RH, and D.

Method 2 -- Two prediction equations were developed for proportion of residual pulpwood VBAR in a sawtimber tree:

1. Height independent variables

(7) $$\hat{P}=-0.2752+0.9937\cdot\frac{RH}{PH}+0.1692\cdot\frac{1}{RH}+0.02340\cdot\frac{PH}{SH}$$

$R^2=0.988$, $s_{y\cdot x}=0.0212$, n=802

2. Height and diameter independent variables

(8) $$\hat{P}=-0.3682+0.8055\cdot\frac{RH}{PH}+0.1208\cdot\frac{1}{RH}+0.02729\cdot\frac{PH}{SH}+2.1513\cdot\frac{1}{D}$$

$R^2=0.992$, $s_{y\cdot x}=0.0174$, n=802

where $\hat{P}$ is the predicted proportion of residual pulpwood VBAR and SH is the number of sawtimber sticks. Table 10 in the Appendix gives values of $\hat{P}$ from Equation 7 for various numbers of sawtimber and residual pulpwood sticks.

Predicted residual pulpwood VBAR can be obtained by multiplying either Equation 7 or 8 times Equation 1 or 3, respectively. Tables 11 and 12 in the Appendix show cubic-foot and rough cord VBARs, respectively, for various values of SH and RH based on Equations 7 and 1. Table 11 values were divided by 75 and multiplied by 0.96 to obtain Table 12 values. Tables could also be developed based on Equations 8 and 3 for various values of SH, RH, and D.

For sawtimber trees with residual pulpwood volume, sawtimber cubic-foot VBAR can be obtained by subtracting predicted residual pulpwood VBAR (using Methods 1 or 2) from predicted pulpwood VBAR (Equations 1 or 3).

Validation

The data set used to validate the prediction equations consisted of the other approximately 20% of the trees in each of the 24 stands, yielding a total of 220 pulpwood trees and 144 sawtimber trees. For each VBAR equation, the average relative error as a percent ($\overline{RE}$) was calculated for each state forest and all state forests pooled where

$$\overline{RE} = \sum_{i=1}^{n} RE_i \Big/ n$$

and $RE_i = [(\hat{VBAR}_i - VBAR_i)/VBAR_i] \times 100$, $\hat{VBAR}_i$ and VBAR are the predicted and actual VBARs for the $i^{\underline{th}}$ tree, and n is the number of trees for a state forest or all state forests pooled. The $\overline{RE}$ for all state forests pooled along with the range for the 6 state forests was determined for all VBAR equations.

For 220 trees, $\overline{RE}$ was -0.42% (range: -1.68 to 2.10%) and 0.40% (range: -1.60 to 1.62%) for the pulpwood cubic-foot VBAR equations based on height

(Equation 1) and height and diameter (Equation 3) independent variables, respectively.

For 144 trees, $\overline{RE}$ was -0.019% (range: -1.42 to 2.34%) and -0.34% (range: -1.74 to 1.23%) for the sawtimber cubic-foot VBAR equations based on height (Equation 2) and height and diameter (Equation 4) independent variables, respectively.

For 144 trees, $\overline{RE}$ was -1.15% (range: -3.53 to 1.90%) and -1.30% (range: -3.60 to 5.28%) for the residual pulpwood cubic-foot VBAR equations based on height (Equation 5) and height and diameter (Equation 6) independent variables, respectively (Method 1). $\overline{RE}$ was 0.077% (range: -2.57 to 3.53%) and 0.073% (range: -2.42 to 1.94%) for residual pulpwood cubic-foot VBARs based on multiplying the proportion of residual pulpwood VBAR Equation 7 times the pulpwood VBAR Equation 1 (height independent variables) and on multiplying the proportion of residual pulpwood VBAR Equation 8 times the pulpwood VBAR Equation 3 (height and diameter independent variables), respectively (Method 2).

There was little difference between the relative errors for equations with just height independent variables and equations with height and diameter independent variables. The relative errors were somewhat less variable over the six state forests for the pulpwood, sawtimber, and residual pulpwood (Method 2) equations with height and diameter variables but somewhat more variable for the residual pulpwood (Method 1) equation. Overall, the relative errors for residual pulpwood VBAR models based on Method 2 were somewhat smaller than those based on Method 1.

One sample was taken in each of two aspen stands in Upper Michigan using a BAF-20 prism. For each stand, DBH to the nearest 0.1 in. and merchantable height to the nearest 100-in. stick to an approximately 3.6" minimum top diameter was measured for each "in" tree with DBH$\geq$4.6".

Ten points were selected in Stand 1. DBH varied from 6.2 to 17.4 in., and merchantable height varied from 3 to 7 sticks. The average number of "in" trees per point was 5.2 (range: 3 to 8).

Five points were selected in Stand 2. DBH varied from 5.6 to 14.6 in., and merchantable height varied from 3-6 sticks. The average number of "in" trees per point was 6.8 (range: 3-12).

Volume in rough cords was estimated for both stands using the new pulpwood prediction Equations 1 (height only independent variables) and 3 (height and diameter independent variables), VBARs currently used by the DNR (Table 3) with merchantable height as the independent variable, and Carlson's Formula.

Results for the two samples are shown in Table 13. For Stand 1, our new Equations 1 and 3 and Carlson's Formula yielded estimates 3.1, 2.1, and 3.7% higher than the estimate based on DNR VBARs. For Stand 2, our new Equations 1 and 3 and Carlson's Formula yielded estimates 3.6, 0.3, and 2.0% higher than the estimate based on DNR VBARs.

Table 13. Volume estimates in rough cords per acre for the samples taken from 2 stands in the Upper Peninsula based on DNR VBARs, our Equations 1 and 3, and Carlson's Formula.

Stand	DNR VBARs	Equation 1	Equation 3	Carlson's Formula
1	32.6	33.6	33.3	33.8
2	39.2	40.6	39.3	40.0

As expected, the new pulpwood VBAR formulas yielded estimates higher than the DNR pulpwood VBAR estimates. Our VBARs are larger than the DNR VBARs for each height class except for the 1 stick class (Table 3). Differences for 2, 3, and 8 sticks are larger than differences for 4-7 sticks.

Guidelines for Users

The new pulpwood, sawtimber, and residual pulpwood VBAR prediction equations using DBH and height independent variables were somewhat more accurate than the respective equations using only height independent variables (Tables 1 and 4 and validation results). However, the differences in accuracy between the 2 sets of equations were very small. Thus, the simpler height only VBAR equations are more than adequate for almost all situations.

Validation results indicated that overall the relative errors for residual pulpwood VBAR models based on Method 2 (multiplying the proportion of residual pulpwood VBAR equation times the pulpwood VBAR equation) were somewhat smaller than those based on Method 1 (residual pulpwood VBAR equation). However, there was very little difference between the 2 methods, and either could be used to estimate residual pulpwood VBAR. A strong argument can be made for Method 2. The use of Method 2 to obtain residual pulpwood VBAR followed by determining sawtimber VBAR as the difference between pulpwood VBAR and residual pulpwood VBAR yields a compatible approach to total VBAR estimation. The use of the sawtimber VBAR and the Method 1 residual pulpwood VBAR equations would not yield the total VBAR obtained by the pulpwood VBAR equation. However, our results show very little difference between these two approaches to estimate total VBAR from sawtimber and residual pulpwood VBAR estimates.

In the long run, our pulpwood rough cord VBARs would yield per acre estimates from 12.8% lower (all trees with 1 stick) to 10.4% higher (all trees with 2 sticks) than estimates based on the VBARs presently used by the DNR. In general, estimates from our VBARs would overestimate by about 8-10% for stands with the number of sticks varying from 2-3, and by about 3-4% for stands with number of sticks varying from 4-7, and by at least 7% for stands with number of sticks 8 or greater.

Carlson's Formula is commonly used to estimate pulpwood rough cord volume per acre. Assume that all of the trees in a prism sample had the same merchantable height. Carlson's Formula would yield estimates 33.3, -5.7, -7.4, -4.6, -1.3, 1.7, 4.7, 7.7, 8.9, and 10.0% larger than estimates based on our new VBARs for 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 sticks, respectively. Carlson's Formula (1) overestimates pulpwood volume for 1-stick trees and trees with 7 or more sticks, and (2) underestimates pulpwood volume for 2- to 4-stick trees compared to our new VBARs. The differences between the 2 methods are small for 5- and 6-stick trees.

We recommend the use of the following cubic-foot VBAR prediction equations using height independent variables for most cruising situations:

1. Pulpwood VBAR - Equation 1 (Table 2, rough cord VBARs-Table 3).
2. Sawtimber VBAR - Equation 2 (Table 2).
3. Sawtimber and residual pulpwood VBARs - Method 2.
    A. Residual pulpwood cubic-foot VBAR - Equation 7 x Equation 1 (Table 11).
    B. Residual pulpwood rough cord VBAR - Convert cubic-foot VBARs to rough cord VBARs (Table 12).
    C. Sawtimber cubic-foot VBAR - subtract the product of Equation 7 and Equation 1 from Equation 1 (subtract the residual pulpwood value in Table 11 from the pulpwood value in Table 1).

For those situations where somewhat more accuracy is needed and the extra cost of measuring DBH is justified, we recommend use of the following cubic-foot VBAR equations using height and DBH independent variables:

1. Pulpwood VBAR - Equation 3 (Table 5, rough cord VBARs-Table 7).
2. Sawtimber VBAR - Equation 4 (Table 6).

3. Sawtimber and residual pulpwood VBAR - Method 2.

   A. Residual pulpwood cubic-foot VBAR (Equation 8 x Equation 3).

   B. Sawtimber cubic-foot VBAR (subtract the product of Equation 8 and Equation 3 from Equation 3).

Literature Cited

Avery, T. E. and H. E. Burkhart. 1983. Forest Measurements. McGraw-Hill Book Co., N.Y. p. 32-36.

Fowler, G. W. and N. G. Hussain. 1987. Bark factor equation for aspen. Michigan DNR Forestry Information Leaflet 2-87. 2 p.

Taras, M. A. 1956. Buying pulpwood by weight as compared with volume measure. U.S. Forest Serv. S.E. Forest Expt. Sta. Paper 74. 11 p.

Appendix

Table 5. VBAR table showing pulpwood cu.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	4.91	12.83	17.94						
6	4.36	12.28	17.38	21.79					
7	3.85	11.77	16.88	21.28	25.40				
8	3.38	11.30	16.40	20.81	24.93	28.91			
9	2.92	10.84	15.95	20.35	24.48	28.46	32.36		
10	2.48	10.41	15.51	19.92	24.04	28.02	31.92		
11		9.98	15.08	19.49	23.61	27.59	31.49	35.34	
12		9.56	14.67	19.07	23.19	27.17	31.07	34.92	
13		9.15	14.25	18.66	22.78	26.76	30.66	34.51	
14		8.74	13.84	18.25	22.37	26.35	30.25	34.10	
15			13.44	17.84	21.97	25.95	29.85	33.70	37.52
16			13.04	17.44	21.56	25.55	29.45	33.30	37.11
17			12.64	17.04	21.17	25.15	29.05	32.90	36.72
18			12.24	16.65	20.77	24.75	28.65	32.50	36.32
19			11.85	16.25	20.37	24.36	28.26	32.11	35.93
20			11.46	15.86	19.98	23.96	27.86	31.72	35.53
21				15.47	19.59	23.57	27.47	31.32	35.14
22				15.08	19.20	23.18	27.08	30.93	34.75
23				14.69	18.81	22.79	26.69	30.54	34.36
24				14.30	18.42	22.40	26.30	30.16	33.97
25				13.91	18.03	22.02	25.92	29.77	33.58

Table 6. VBAR table showing sawtimber cu.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
9	7.68	14.43	20.08	25.46	30.72				
10	7.09	13.85	19.50	24.87	30.14	35.35			
11	6.34	13.10	18.75	24.13	27.39	34.60			
12	5.48	12.23	17.89	23.26	28.53	33.74	38.92		
13	4.52	11.27	16.93	22.30	27.57	32.78	37.96	43.12	
14	3.49	10.24	15.89	21.27	26.54	31.75	36.93	42.08	
15	2.40	9.15	14.80	20.18	25.45	30.66	35.83	40.99	46.14
16		8.01	13.67	19.04	24.31	29.52	34.70	39.86	45.00
17		6.84	12.49	17.87	23.13	28.34	33.52	38.68	43.83
18		5.63	11.28	16.66	21.92	27.13	32.31	37.47	42.62
19		4.39	10.04	15.42	20.69	25.90	31.07	36.23	41.38
20		3.13	8.78	14.16	19.43	24.64	29.81	34.97	40.12
21			7.50	12.88	18.15	23.36	28.53	33.69	38.84
22			6.21	11.58	16.85	22.06	27.24	32.40	37.54
23			4.89	10.27	15.54	20.75	25.93	31.08	36.23
24			3.57	8.95	14.21	19.42	24.60	29.76	34.91
25			2.23	7.61	12.87	18.08	23.26	28.42	33.57

Table 7. VBAR table showing rough cords/sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	0.065	0.164	0.230						
6	0.057	0.157	0.223	0.275					
7	0.051	0.151	0.216	0.269	0.317				
8	0.044	0.145	0.210	0.263	0.311	0.356			
9	0.038	0.139	0.204	0.257	0.305	0.350	0.393		
10	0.033	0.133	0.199	0.252	0.300	0.345	0.388		
11		0.128	0.193	0.246	0.294	0.340	0.383	0.424	
12		0.122	0.188	0.241	0.289	0.334	0.378	0.419	
13		0.117	0.182	0.236	0.284	0.329	0.373	0.414	
14		0.112	0.177	0.230	0.279	0.324	0.368	0.409	
15			0.172	0.225	0.274	0.319	0.363	0.404	0.450
16			0.167	0.220	0.269	0.314	0.358	0.400	0.445
17			0.162	0.215	0.264	0.310	0.353	0.395	0.441
18			0.157	0.210	0.259	0.305	0.348	0.390	0.436
19			0.152	0.205	0.254	0.300	0.343	0.385	0.431
20			0.147	0.200	0.249	0.295	0.339	0.381	0.426
21				0.195	0.244	0.290	0.334	0.376	0.422
22				0.190	0.239	0.285	0.329	0.371	0.417
23				0.186	0.235	0.281	0.324	0.367	0.412
24				0.181	0.230	0.276	0.320	0.362	0.408
25				0.176	0.225	0.271	0.315	0.357	0.403

Table 8. Residual pulpwood cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 5.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	8.64	10.19	13.19	16.33	19.48	22.60	25.71	28.80
2	5.80	7.36	10.64	14.06	17.45	20.78	24.05	27.28
3	4.38	5.66	8.94	12.44	15.93	19.36	22.73	26.04
4	3.53	4.52	7.72	11.23	14.75	18.23	21.65	25.01
5	2.96	3.71	6.81	10.28	13.80	17.30	20.74	24.14
6	2.56	3.10	6.10	9.52	13.03	16.52	19.98	23.39
7	2.25	2.63	5.53	8.91	12.39	15.87	19.32	22.74
8	2.02	2.25	5.07	8.39	11.84	15.31	18.76	22.18

Table 9. Residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 5.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.111	0.130	0.169	0.209	0.249	0.289	0.329	0.369
2	0.074	0.094	0.136	0.180	0.223	0.266	0.308	0.349
3	0.056	0.072	0.114	0.159	0.204	0.248	0.291	0.333
4	0.045	0.058	0.099	0.144	0.189	0.233	0.277	0.320
5	0.038	0.048	0.087	0.132	0.177	0.221	0.266	0.309
6	0.033	0.040	0.078	0.122	0.167	0.211	0.256	0.299
7	0.029	0.034	0.071	0.114	0.159	0.203	0.247	0.291
8	0.026	0.029	0.065	0.107	0.152	0.196	0.240	0.284

Table 10. Proportions of residual pulpwood cubic-foot VBAR in a sawtimber tree for various numbers of sawtimber and residual pulpwood sticks based on Equation 7.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.44	0.54	0.62	0.68	0.73	0.77	0.81	0.84
2	0.26	0.35	0.44	0.50	0.55	0.59	0.63	0.66
3	0.17	0.25	0.32	0.39	0.44	0.49	0.52	0.55
4	0.12	0.18	0.25	0.31	0.36	0.41	0.45	0.48
5	0.09	0.13	0.19	0.25	0.30	0.35	0.38	0.42
6	0.06	0.09	0.15	0.20	0.25	0.30	0.33	0.37
7	0.04	0.06	0.11	0.17	0.21	0.26	0.29	0.33
8	0.03	0.04	0.08	0.13	0.18	0.22	0.26	0.29

Table 11. Residual pulpwood cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equations 7 and 1.

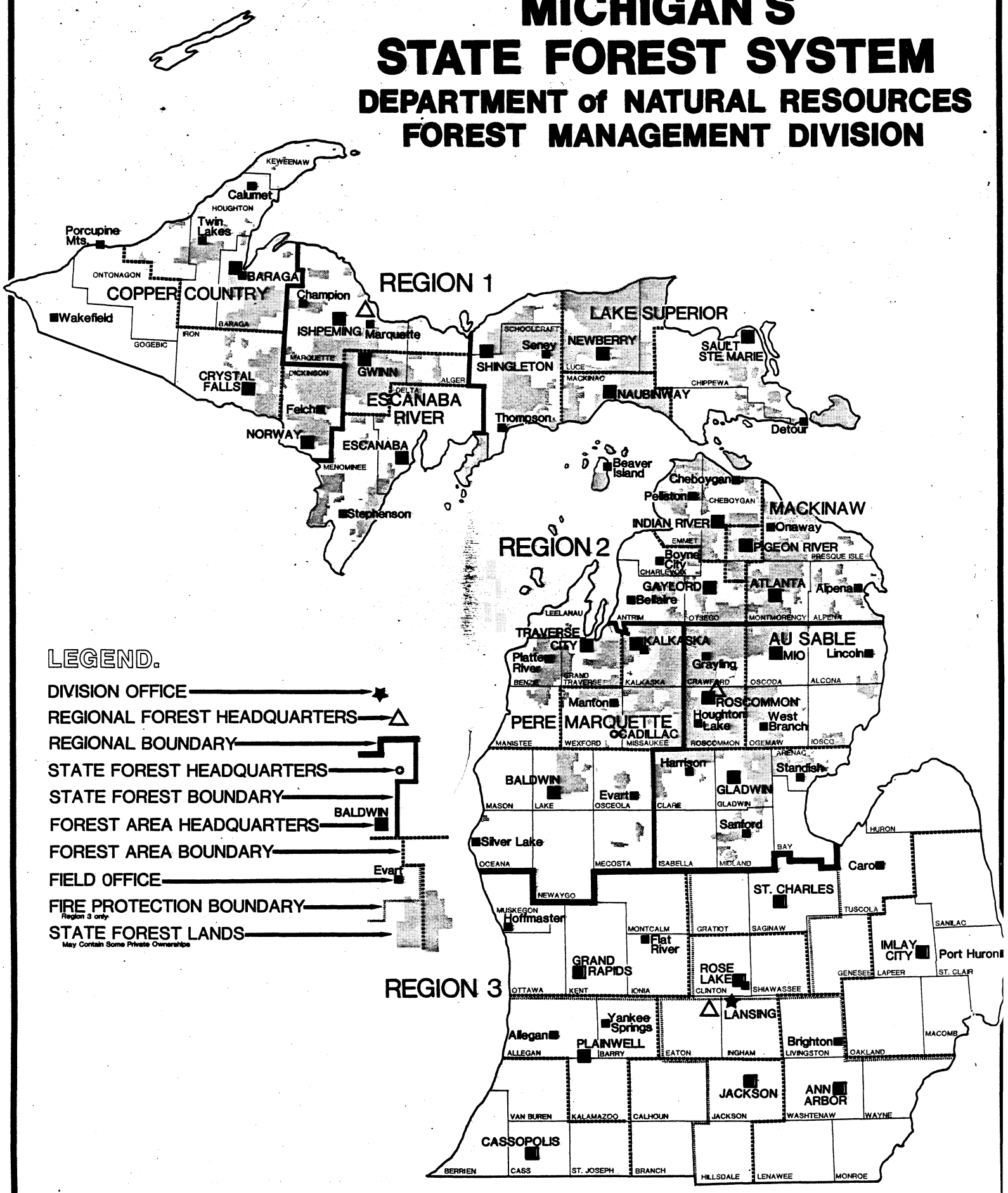
Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	5.43	9.13	12.85	16.56	20.31	24.13	28.07	32.12
2	4.39	7.32	10.63	13.96	17.28	20.62	23.99	27.40
3	3.60	6.00	9.07	12.23	15.40	18.58	21.76	24.96
4	2.97	4.91	7.79	10.83	13.90	16.98	20.06	23.16
5	2.45	3.96	6.66	9.60	12.59	15.60	18.62	21.65
6	1.99	3.10	5.64	8.48	11.40	14.35	17.32	20.30
7	1.57	2.31	4.69	7.44	10.30	13.20	16.12	19.06
8	1.18	1.56	3.80	6.46	9.26	12.11	15.00	17.90

Table 12. Residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equations 7 and 1.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.070	0.117	0.165	0.212	0.260	0.309	0.359	0.411
2	0.056	0.094	0.136	0.179	0.221	0.264	0.307	0.351
3	0.046	0.077	0.116	0.157	0.197	0.238	0.279	0.320
4	0.038	0.063	0.100	0.139	0.178	0.217	0.257	0.296
5	0.031	0.051	0.085	0.123	0.161	0.200	0.238	0.277
6	0.025	0.040	0.072	0.109	0.146	0.184	0.222	0.260
7	0.020	0.030	0.060	0.095	0.132	0.169	0.206	0.244
8	0.015	0.020	0.049	0.083	0.118	0.155	0.192	0.229

MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES
FOREST MANAGEMENT DIVISION



LEGEND.

- DIVISION OFFICE
- REGIONAL FOREST HEADQUARTERS
- REGIONAL BOUNDARY
- STATE FOREST HEADQUARTERS
- STATE FOREST BOUNDARY
- FOREST AREA HEADQUARTERS (BALDWIN)
- FOREST AREA BOUNDARY
- FIELD OFFICE (Evart)
- FIRE PROTECTION BOUNDARY (Region 3 only)
- STATE FOREST LANDS (May Contain Some Private Ownership)