VOLUME AND VOLUME-BASAL AREA RATIO SAWTIMBER EQUATIONS FOR ASPEN IN MICHIGAN: 9.6-IN. MINIMUM TOP DIAMETER

BY

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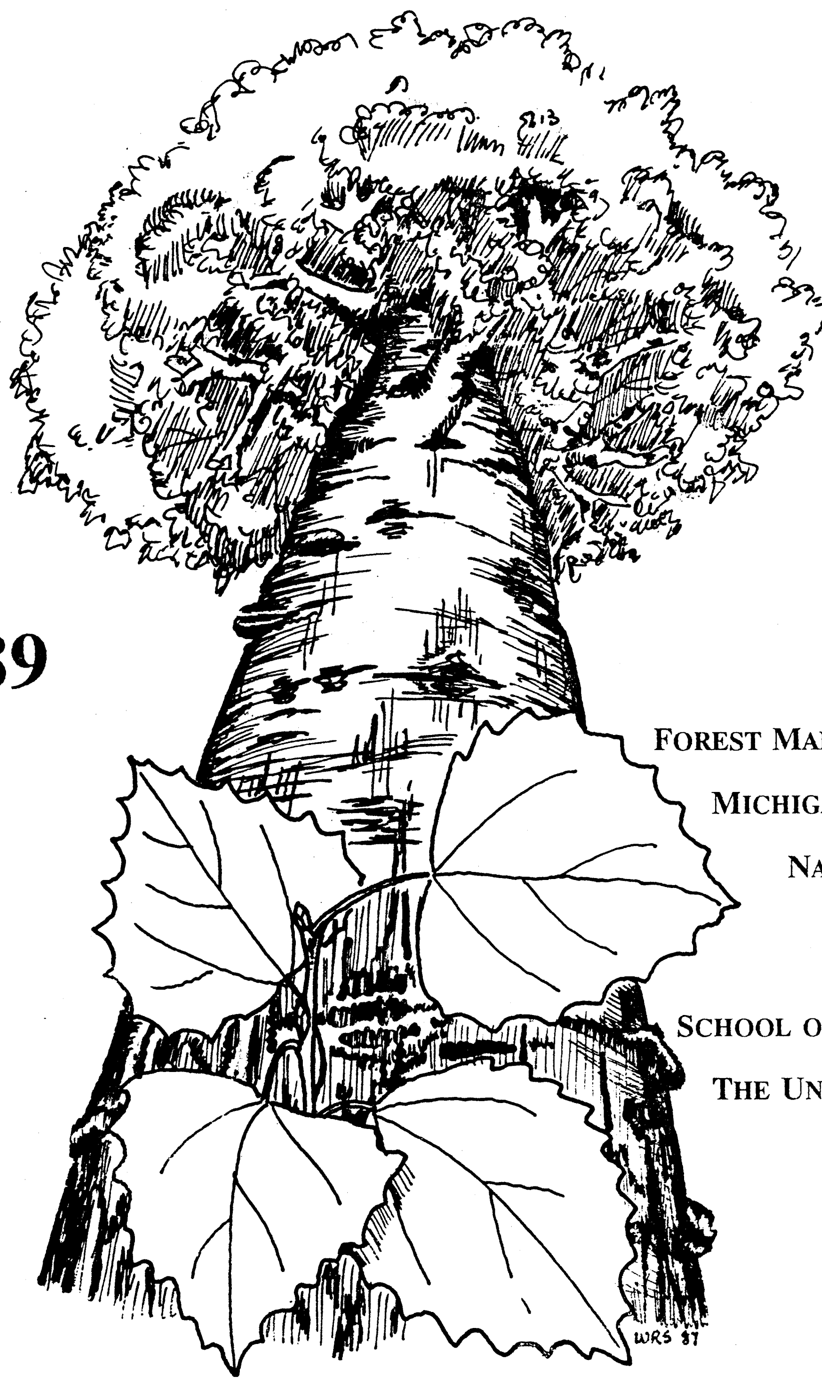

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Management Summary

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot individual tree sawtimber volume and volume-basal area ratio (VBAR in cu.ft./sq.ft. or bd.ft./sq.ft.) equations for merchantable volume to a 9.6-in. minimum inside bark top diameter were developed for aspen in Michigan. Data used to develop these equations were collected from 24 aspen stands in Michigan (12 stands each from the Upper and Lower Peninsulas). Four stands each were sampled from each of the six state forests in Michigan.

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y.x}$), and sample error terms indicated that (1) nonlinear equations were most accurate for all types of individual tree volumes, and (2) linear equations were most accurate for all types of VBARs. VBAR equations using diameter at breast height (DBH) and merchantable height (MH) yielded somewhat more accuracy than VBAR equations using MH independent variables. However, the differences between the 2 sets of equations are relatively small, indicating that the use of the simpler height VBAR equations is justified for most cruising situations.

The new individual tree volume equations yielded, in general, volume estimates larger than the values in Tables 2-5 of Fowler and Hussain (1988a) for a 7.6-in. minimum top diameter with the difference increasing and decreasing with increasing MH and DBH, respectively. The new VBAR equations yielded VBAR estimates larger than the values in Table 2 of Fowler and Hussain (1988b) for a 7.6-in. minimum top diameter with the difference,

in general, increasing and decreasing with increasing MH for cu.ft. and bd.ft. VBARs, respectively.

The new individual tree volume equations are:

1. Cubic-foot volume

$$\hat{V}_C=0.4063+0.1673D^{1.401}H^{0.954}$$

2. Doyle board-foot volume

$$\hat{V}_D=3.121+0.2123D^{1.754}H^{1.131}$$

3. International 1/4-inch board-foot volume

$$\hat{V}_I=4.108+1.053D^{1.300}H^{1.101}$$

4. Scribner board-foot volume

$$\hat{V}_S=3.937+0.8971D^{1.348}H^{1.102}$$

where D is DBH in inches and H is MH in 100-in. sticks to a 9.6-in. top diameter limit.

Multiple linear regression equations were developed to predict (1) one type of volume from another type and (2) Doyle, International 1/4-inch, and Scribner board-foot cubic-foot ratios as a function of D, H, and D and H.

We recommend the use of the following VBAR equations in most cruising situations for aspen:

1. Cubic-foot VBAR

$$\widehat{VBAR}_C=6.288+4.639\cdot H-3.177\cdot\frac{1}{H}$$

2. Doyle board-foot VBAR

$$\widehat{VBAR}_D=0.4726+24.971\cdot H+0.1822\cdot\frac{1}{H}$$

3. International 1/4-inch board-foot VBAR

$$\hat{VBAR}_I=23.932+30.740\cdot H-13.796\cdot\frac{1}{H}$$

4. Scribner board-foot VBAR

$$\hat{VBAR}_S=20.776+30.282\cdot H-11.960\cdot\frac{1}{H}$$

Multiple linear regression equations were also developed to predict one type of VBAR from another for the 4 types of VBARs examined in this study.

The above equations can be used to develop tables as we have done in the paper or entered into a computer program to facilitate computer volume calculations for cruise data.

Michigan Department of Natural Resources
Forest Management Division

SUBJECT -- INDIVIDUAL TREE VOLUME AND VOLUME-BASAL AREA RATIO EQUATIONS

DATE -- 16 Dec. 88

TITLE -- Volume and Volume-Basal Area Ratio Sawtimber Equations for Aspen in Michigan: 9.6-in. Minimum Top Diameter

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Background

Composite individual tree sawtimber board-foot (i.e., Scribner and International 1/4-inch rules) volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955). The Michigan Department of Natural Resources (MDNR) developed International 1/4-inch sawtimber volume-basal area ratios (VBARs) in bd.ft. per sq.ft. to be used in prism cruising (DNR Tally Sheet R 4145). Fowler and Hussain (1988a,b) developed cu.ft. and bd.ft. (Doyle, International 1/4-inch, and Scribner) volume and volume-basal area ratio equations and tables for aspen in Michigan. All of the above equations and tables are based on a 7.6 to 8.0-in. minimum top diameter.

Purpose

The MDNR needs sawtimber volume and volume-basal area equations and tables for aspen in Michigan based on a 9.6-in. minimum top diameter. The purpose of this paper is to present such equations for sawtimber cubic feet and Doyle, International 1/4-inch, and Scribner board feet.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 1381 trees from 24 stands as follows:

1) 677 trees from 12 stands in the Upper Peninsula (i.e., 4 stands each in the Copper, Escanaba River, and Lake Superior state forests), and
2) 704 trees from 12 stands in the Lower Peninsula (i.e., 4 stands each in the Mackinaw, AuSable, and Marquette state forests).

Measurements were taken on 728 bigtooth aspen and 653 trembling aspen trees. Stands were selected from the 6 state forests to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1986.

For the 24 stands, site index varied from 51 to 79, age varied from 47 to 70 years, basal area/acre varied from 70-186 sq.ft., average DBH varied from 7.7 to 11.9 in., average total height (TH) varied from 52.2 to 77.5 ft., and average merchantable height (MH) to an approximate 3.6-in. minimum top diameter varied from 3.4 to 7.7 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., TH to the nearest ft., MH to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b,c) developed pulpwood, sawtimber, and residual pulpwood cu.ft. volume and VBAR equations, respectively, and Fowler and Hussain (1988a,b) developed sawtimber cu.ft. and bd.ft. volume and VBAR equations for a 7.6-in. minimum top diameter, respectively, from the total data set described above.

For this paper, sawtimber trees were defined as trees that had at least one 100-in. stick with a minimum top diameter no smaller than 9.6 inches. Sawtimber MH is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 9.6 inches. There was a total of 398 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into 2 pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the 2 volumes. For each 100-in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V=\frac{(B+b)L}{2}$ (Avery and Burkhart 1983)

Doyle: $V=0.5D^2-4.0D+8.0$ (Husch et al. 1982)

International 1/4-inch: $V=0.905(0.44D^2-1.20D-0.30)$ (Husch et al. 1982)

Scribner: $V=0.395D^2-0.99D-2.15$ (Bruce and Schumacher 1950)

where

V=volume in cubic feet or board feet,

L=length of stick (100-in.) in ft.,

B=cross-sectional area inside bark of large end of the stick in sq.ft.,

b=cross-sectional area inside bark of small end of the stick in sq.ft., and

D=diameter of small end of the stick inside bark in inches.

See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the 3 board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 9.6-in. top diameter limit. Sawtimber VBARs were obtained for each tree by dividing the 4 volumes by the basal area in sq.ft. of the tree at 4.5 ft. above the ground.

Individual tree volume was regressed on various variables determined from tree DBH and MH using multiple linear and non-linear regression. VBAR was regressed on various forms of DBH and MH using multiple linear regression.

Results

Fowler and Hussain (1987b) found no significant difference between pulpwood cubic-foot volume equations of the 6 forest areas and 2 species, so the data for both species and all stands were pooled before developing volume and VBAR equations. All equations in this paper were based on the 398 sawtimber trees with an average DBH=12.2 in. (range: 10.2 to 16.7), average MH=2.0 sticks (range: 1 to 6), average cubic-foot volume=11.8 (range: 4.6 to 50.4), average cubic-foot VBAR=13.6 cu.ft./sq.ft. (range: 6.8 to 33.5), average Doyle bd.ft. volume=45.1 (range: 15.4 to 251.5), average Doyle bd.ft. VBAR=51.0 bd.ft./sq.ft.

(range: 21.1 to 167.3), average International 1/4-in. bd.ft. volume=67.6 (range: 25.7 to 324.1), average International 1/4-in. bd.ft. VBAR=77.2 bd.ft./sq.ft. (range: 33.4 to 215.7), average Scribner bd.ft. volume=65.1 (range: 24.4 to 317.1), and average Scribner bd.ft. VBAR=74.3 bd.ft./sq.ft. (range: 32.0 to 211.0).

Individual Tree Volume Equations

Cubic-foot and board-foot volume prediction equations

A comparison of various multiple linear regression and non-linear regression equations based on goodness-of-fit and simplicity indicated that the following nonlinear prediction equation compared favorably to all other equations examined for cubic-foot and the three board-foot volumes:

$$\hat{V} = \hat{\beta}_0+\hat{\beta}_1 D^{\hat{\beta}_2} H^{\hat{\beta}_3}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is MH in 100-in sticks to a 9.6-in. top diameter limit. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ are the sample regression coefficients related to the independent variables.

Table 1 shows the sawtimber volume prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot volumes along with standard errors of the estimate ($s_{y \cdot x}$) and coefficients of determination (R^2).

A cubic-foot volume table is shown in Table 2, and Doyle, International 1/4-inch, and Scribner board-foot volume tables are shown in Tables 3, 4, and 5, respectively.

Table 1. Estimated parameters ($\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$), standard errors of the estimate ($s_{y\cdot x}$) and coefficients of determination (R^2) for cubic-foot and three board-foot volumes.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$s_{y\cdot x}$	R^2
(1) Cubic-foot[a]	0.4063	0.1673	1.401	0.9540	0.59	0.994
(2) Doyle[b]	3.121	0.2123	1.754	1.131	4.14	0.985
(3) International 1/4-inch[c]	4.108	1.053	1.300	1.101	4.52	0.991
(4) Scribner[d]	3.937	0.8971	1.348	1.102	4.52	0.990

[a] $\hat{V}=0.4063+0.1673D^{1.401}H^{0.9540}$

[b] $\hat{V}=3.121+0.2123D^{1.754}H^{1.131}$

[c] $\hat{V}=4.108+1.053D^{1.300}H^{1.101}$

[d] $\hat{V}=3.937+0.8971D^{1.348}H^{1.102}$

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Table 2. Volume table showing cu.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 1).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	5.2	9.7	14.1	18.5	22.8	27.0			
12	5.8	10.9	15.9	20.8	25.7	30.5	35.2		
13	6.5	12.2	17.8	23.2	28.7	34.0	39.3	44.6	
14	7.2	13.5	19.7	25.7	31.7	37.7	43.6	49.5	
15	7.8	14.8	21.6	28.3	34.9	41.5	48.0	54.5	60.9
16		16.2	23.6	30.9	38.2	45.4	52.5	59.6	66.6
17		17.6	25.7	33.7	41.5	49.4	57.1	64.8	72.5
18		19.0	27.8	36.4	45.0	53.4	61.8	70.2	78.5
19		20.5	29.9	39.3	48.5	57.6	66.7	75.7	84.6
20		22.0	32.1	42.2	52.1	61.9	71.6	81.3	90.9
21			34.4	45.1	55.7	66.2	76.6	87.0	97.3
22			36.7	48.1	59.4	70.6	81.8	92.8	103.8
23			39.0	51.2	63.2	75.2	87.0	98.8	110.5
24			41.4	54.3	67.1	79.8	92.3	104.8	117.2
25			43.8	57.5	71.0	84.4	97.7	111.0	124.1

Table 3. Volume table showing Doyle bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 2).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	17	34	52	71	91	111			
12	20	39	61	83	106	129	153		
13	22	45	69	95	121	148	176	204	
14	25	51	78	107	137	168	199	231	
15	28	57	88	121	155	189	225	261	298
16		63	98	135	173	212	251	292	333
17		70	109	150	192	235	279	324	370
18		77	120	165	212	259	308	358	409
19		84	132	181	232	285	339	393	449
20		92	144	198	254	311	370	430	491
21			156	215	276	339	403	468	534
22			170	234	300	368	437	508	580
23			183	252	324	397	472	549	626
24			197	272	349	428	509	591	675
25			211	291	374	459	546	635	725

Table 4. Volume table showing International 1/4-inch bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 3).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	28	55	84	114	144	175			
12	31	61	93	127	161	196	231		
13	34	67	103	140	178	217	256	296	
14	37	74	113	154	196	238	281	325	
15	40	80	123	168	213	260	307	355	404
16		87	134	182	232	282	334	386	439
17		94	144	197	250	305	361	417	475
18		101	155	212	269	328	388	449	511
19		108	166	227	289	352	416	482	548
20		115	178	242	308	376	445	515	585
21			189	258	328	400	474	548	623
22			200	274	349	425	503	582	662
23			212	290	369	450	533	616	701
24			224	306	390	476	563	651	741
25			236	322	411	501	593	687	781

Table 5. Volume table showing Scribner bd.ft. volume for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 4).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	27	53	80	109	138	168			
12	29	59	90	122	155	188	222		
13	32	65	99	135	172	209	247	286	
14	35	71	110	149	189	231	273	315	358
15	38	78	120	163	207	253	299	345	393
16		85	130	178	226	275	326	377	428
17		92	141	192	245	298	353	408	464
18		99	152	207	264	322	381	441	501
19		106	163	223	284	346	409	474	539
20		113	175	238	304	371	438	507	577
21			186	254	324	395	468	541	616
22			198	271	345	421	498	576	656
23			210	287	366	446	528	612	696
24			222	304	387	473	559	647	737
25			235	321	409	499	591	684	778

The values shown in Table 2 (cu.ft. volumes) are larger than the values in Table 2 of Fowler and Hussain (1988a), varying from 13-15% higher for DBH=11 in. to about 5% higher for DBH=25 in.

The values shown in Table 3 (Doyle bd.ft. volumes) are, in general, larger than the values in Table 3 of Fowler and Hussain (1988a), varying from 42-44% higher for DBH=11 in. to 2-5% lower for DBH=25 in.

The values shown in Table 4 (International 1/4-inch bd.ft. volumes) are larger than those values in Table 4 of Fowler and Hussain (1988a), varying from 27 to 31% higher for DBH=11 in. to 0-3% higher for DBH=25 in.

The values shown in Table 5 (Scribner bd.ft. volumes) are larger than those values in Table 5 of Fowler and Hussain (1988a), varying from 29-33% higher for DBH=11 in. to 0-1% for DBH=25 in.

For all 4 types of volumes, values for a 9.6-in. minimum top diameter were, in general, larger than values for a 7.6-in. minimum top diameter, with the difference increasing and decreasing with increasing MH and DBH, respectively.

Predicting One Type of Volume From Another

Multiple linear regression equations were developed to predict one type of volume from another using the 398 sawtimber trees. Equations were developed for predicting cubic-foot volume (CV) as a function of Doyle (DV), International 1/4-inch (IV), and Scribner (SV) bd.ft. volumes, DV as a function of CV, IV, and SV, IV as a function of CV, DV, and SV, and SV as a function of

CV, DV, and IV. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 6.

Predicting Board-foot Cubic-foot Ratios

Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios were calculated for each of the 398 sawtimber trees. Average board-foot cubic-foot ratios were 3.62 (range: 2.72 to 4.99), 5.56 (range: 4.44 to 6.43), and 5.34 (range: 4.23 to 6.29) for Doyle, International 1/4-inch, and Scribner board-foot volumes, respectively.

Multiple linear regression equations were developed to predict the three types of board-foot cubic-foot ratios as a function of D, H, and D and H. These equations and their associated values of R^2 and $s_{y \cdot x}$ are shown in Table 7.

Board-foot cubic-foot ratios for the three types of board-foot volumes as a function of H and D are shown in Tables 8 and 9, respectively. The values in Table 8 for a 9.6-in. top diameter limit are larger than the values in Table 8 of Fowler and Hussain (1988a) for a 7.6-in. top diameter limit, with the difference decreasing with increasing H for all three types of volume. The values in Table 9 for a 9.6-in. top diameter limit vary from 4 to 14% larger for D=11 in., 0 to 5% larger for D=15 in., 2 to 3% smaller for D=20 in., and 4-5% smaller for D=25 in. compared to the values in Table 9 of Fowler and Hussain (1988a) for a 7.6-in. top diameter limit.

Table 6. Regression equations for predicting cubic-foot (CV) and Doyle (DV), International 1/4-inch (IV), and Scribner (SV) board-foot volumes from the other three types of volumes.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CV}=0.8899+0.2614DV-0.000264DV^2$	0.996	0.51
$\hat{CV}=0.5161+0.1730IV-0.000054IV^2$	0.997	0.44
$\hat{CV}=0.5556+0.1799SV-0.000067SV^2$	0.997	0.44
$\hat{DV}=-1.6480+3.4631CV+0.029193CV^2$	0.996	2.24
$\hat{DV}=-0.6363+0.6310IV+0.000466IV^2$	0.999	1.13
$\hat{DV}=-0.5761+0.6598SV+0.000439SV^2$	0.999	0.99
$\hat{IV}=-2.3237+5.6649CV+0.014117CV^2$	0.997	2.66
$\hat{IV}= 1.8735+1.5309DV-0.001066DV^2$	0.999	1.58
$\hat{IV}= 0.1951+1.0413SV-0.000066SV^2$	1.000	0.19
$\hat{SV}=-2.3461+5.4230CV+0.016239CV^2$	0.997	2.60
$\hat{SV}= 1.5426+1.4743DV-0.000936DV^2$	0.999	1.35
$\hat{SV}=-0.1783+0.9600IV+0.000061IV^2$	1.000	0.19

Table 7. Regression equations for predicting Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D, H, and D and H.

Type of Ratio	Regression Equation	R^2	$s_{y \cdot x}$
	$\hat{DCR}=0.603+0.247D$	0.576	0.24
Doyle	$\hat{DCR}=3.043+0.284H$	0.690	0.21
	$\hat{DCR}=2.377+0.0641D+0.227H$	0.701	0.20
	$\hat{ICR}=3.685+0.154D$	0.242	0.31
International 1/4-inch	$\hat{ICR}=5.103+0.226H$	0.475	0.26
	$\hat{ICR}=6.167-0.102D+0.317H$	0.505	0.25
	$\hat{SCR}=3.244+0.172D$	0.294	0.29
Scribner	$\hat{SCR}=4.857+0.240H$	0.517	0.25
	$\hat{SCR}=5.649-0.0763D+0.307H$	0.534	0.25

Table 8. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of H.

H	Board-foot Cubic-foot Ratio		
(sticks)	DCR	ICR	SCR
1	3.3	5.3	5.1
2	3.6	5.6	5.3
3	3.9	5.8	5.6
4	4.2	6.0	5.8
5	4.5	6.2	6.1
6	4.7	6.5	6.3
7	5.0	6.7	6.5
8	5.3	6.9	6.8
9	5.6	7.1	7.0
10	5.9	7.4	7.3

Table 9. Predicted Doyle (DCR), International 1/4-inch (ICR), and Scribner (SCR) board-foot cubic-foot ratios as a function of D.

D (inches)	Board-foot Cubic-foot Ratio		
	DCR	ICR	SCR
11	3.3	5.4	5.1
12	3.6	5.5	5.3
13	3.8	5.7	5.5
14	4.1	5.8	5.7
15	4.3	6.0	5.8
16	4.6	6.1	6.0
17	4.8	6.3	6.2
18	5.0	6.5	6.3
19	5.3	6.6	6.5
20	5.5	6.8	6.7
21	5.8	6.9	6.9
22	6.0	7.1	7.0
23	6.3	7.2	7.2
24	6.5	7.4	7.4
25	6.8	7.5	7.5

VBAR Equations

Cubic-foot and board-foot equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated the following prediction equations compared favorably to all other equations examined for cubic-foot and the 3 board-foot VBARs:

1. Height independent variables

$$\hat{VBAR}=\hat{\beta}_0+\hat{\beta}_1 H+\hat{\beta}_2 \frac{1}{H}$$

2. Height and DBH independent variables

$$\hat{VBAR}=\hat{\beta}_0+\hat{\beta}_1 H+\hat{\beta}_2 \frac{1}{H}+\hat{\beta}_3 D+\hat{\beta}_4 \frac{1}{D}$$

where $\hat{VBAR}$ is predicted VBAR, H is MH in 100-in. sticks to a 9.6-in. top diameter limit, and D is DBH in inches. $\hat{\beta}_0$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, $\hat{\beta}_4$ are the sample regression coefficients related to the independent variables.

Table 10 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs and their associated R^2 and $s_{y \cdot x}$ values for the height only models. Table 11 shows the cubic-foot and 3 board-foot VBARs for various MHs based on Equations 5-8. The values in Table 11 for a 9.6-in. minimum top diameter are larger than the values in Table 2 of Fowler and Hussain (1988b) for a 7.6-in. minimum top diameter with the difference increasing with height for cu.ft. ratios (from 0.4 to 7.4%) and decreasing with height

Table 10. Estimated intercepts ($\hat{\beta}_0$), regression coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$, and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and three board-foot VBAR prediction equations with height independent variables.

Prediction Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	$s_{y \cdot x}$
(5) Cubic-foot[a]	6.288	4.639	- 3.177	0.982	0.79
(6) Doyle[b]	0.4726	24.971	0.1822	0.977	4.15
(7) International 1/4-inch[c]	23.932	30.740	-13.796	0.978	5.63
(8) Scribner[d]	20.776	30.282	-11.960	0.978	5.44

[a] $\hat{VBAR}=6.288+4.639 \cdot H-3.177 \cdot \frac{1}{H}$

[b] $\hat{VBAR}=0.4726+24.971 \cdot H+0.1822 \cdot \frac{1}{H}$

[c] $\hat{VBAR}=23.932+30.740 \cdot H-13.796 \cdot \frac{1}{H}$

[d] $\hat{VBAR}=20.776+30.282 \cdot H-11.960 \cdot \frac{1}{H}$

Table 11. VBARs in cu.ft./sq.ft. or bd.ft./sq.ft. for the four types of volume for various values of MH.

MH (sticks)	Cu.ft./Sq.ft.	Bd.ft./Sq.ft. Doyle	Scribner	Int. 1/4-inch
1	7.75	26	39	41
2	13.98	51	75	79
3	19.15	75	108	112
4	24.05	100	139	143
5	28.85	125	170	175
6	33.59	150	200	206
7	38.31	175	231	237
8	43.00	200	262	268
9	47.69	225	292	299
10	52.36	250	322	330

for Doyle (from 45.7 to 20.8%), International 1/4-inch (from 13.9 to 11.6%), and Scribner (from 18.2 to 12.6%) bd.ft. ratios.

Table 12 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs for the height and diameter models. Note that R^2 and $s_{y\cdot x}$ for these equations are somewhat larger and smaller, respectively, than for the respective equations based on height only (Table 10). A sawtimber cubic-foot VBAR table based on Equation 9 is shown in Table 13, and Doyle, International 1/4-inch, and Scribner board-foot VBAR tables are shown in Tables 14, 15, and 16 based on Equations 10, 11, and 12, respectively.

Predicting One Type of VBAR From Another

Multiple linear regression equations were developed to predict one type of VBAR from another using the 398 sawtimber trees. Equations were developed for predicting cubic-foot VBAR (CR) as a function of Doyle (DR), International 1/4-inch (IR), and Scribner (SR) VBARs, DR as a function of CR, IR, and SR, IR as a function of CR, SR, and DR, and SR as a function of CR, DR, and IR. These equations and their associated R^2 and $s_{y\cdot x}$ values are shown in Table 17.

Guidelines for Users

We recommend the use of Equations 1-4 for estimating individual tree volumes to a 9.6-in. minimum top diameter for aspen in Michigan for fixed-area plot sampling. We also recommend the use of Equations 5-8 for estimating VBARs to a 9.6-in. minimum top diameter for aspen in Michigan for prism cruising in most situations. For those situations where somewhat

Table 12. Estimated intercepts ($\hat{\beta}_0$), regression coefficients $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$ and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and three board-foot VBAR prediction equations with independent variables based on height and diameter.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2	$s_{y \cdot x}$
(9) Cubic-foot[a]	55.741	5.675	-1.763	- 2.462	- 271.523	0.989	0.64
(10) Doyle[b]	70.530	26.977	2.273	- 3.835	- 347.446	0.978	4.04
(11) International 1/4-inch[c]	281.796	37.621	-6.181	-13.791	-1313.650	0.986	4.51
(12) Scribner[d]	259.106	36.538	-4.939	-12.679	-1221.315	0.985	4.51

[a] $\hat{VBAR}=55.741+5.675 \cdot H-1.763 \cdot \frac{1}{H}-2.462 \cdot D-271.523 \cdot \frac{1}{D}$

[b] $\hat{VBAR}=70.530+26.977 \cdot H+2.273 \cdot \frac{1}{H}-3.835 \cdot D-347.446 \cdot \frac{1}{D}$

[c] $\hat{VBAR}=281.796+37.621 \cdot H-6.181 \cdot \frac{1}{H}-13.791 \cdot D-1313.650 \cdot \frac{1}{D}$

[d] $\hat{VBAR}=259.106+36.538 \cdot H-4.939 \cdot \frac{1}{H}-12.679 \cdot D-1221.315 \cdot \frac{1}{D}$

Table 13. VBAR showing cu.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 9).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	7.89	14.44	20.41	26.23	32.00	37.73			
12	7.48	14.04	20.01	25.83	31.59	37.33	43.04		
13	6.76	13.32	19.29	25.11	30.87	36.60	42.32	48.03	
14	5.79	12.35	18.32	24.14	29.90	35.63	41.35	47.06	
15	4.62	11.18	17.15	22.97	28.73	34.47	40.18	45.89	51.59
16		9.85	15.82	21.64	27.40	33.13	38.85	44.56	50.26
17		8.38	14.35	20.17	25.94	31.67	37.39	43.09	48.79
18		6.81	12.78	18.60	24.36	30.10	35.81	41.52	47.22
19		5.14	11.11	16.93	22.69	28.43	34.15	39.85	45.55
20		3.39	9.36	15.18	20.95	26.68	32.40	38.10	43.80
21			7.55	13.37	19.13	24.87	30.58	36.29	41.99
22			5.67	11.49	17.26	22.99	28.71	34.41	40.11
23			3.75	9.57	15.33	21.07	26.78	32.49	38.19
24			1.78	7.60	13.36	19.10	24.81	30.52	36.22
25				5.59	11.35	17.09	22.80	28.51	34.21

Table 14. VBAR table showing Doyle bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 10).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	26.0	51.8	78.4	105.2	132.1	159.0			
12	24.8	50.6	77.2	104.0	130.9	157.8	184.7		
13	23.2	49.0	75.6	102.4	129.3	156.2	183.1	210.1	
14	21.3	47.1	73.7	100.5	127.4	154.3	181.2	208.1	
15	19.1	44.9	71.5	98.3	125.2	152.1	179.0	205.9	232.9
16		42.5	69.1	95.9	122.8	149.7	176.6	203.6	230.5
17		40.0	66.6	93.4	120.2	147.1	174.1	201.0	227.9
18		37.3	63.9	90.7	117.5	144.4	171.4	198.3	225.2
19		34.5	61.1	87.9	114.7	141.6	168.5	195.5	222.4
20		31.5	58.1	84.9	111.8	138.7	165.6	192.6	219.5
21			55.1	81.9	108.8	135.7	162.6	189.6	216.5
22			52.1	78.8	105.7	132.6	159.5	186.5	213.4
23			48.9	75.7	102.6	129.5	156.4	183.3	210.3
24			45.7	72.5	99.4	126.3	153.2	180.1	207.1
25			42.4	69.2	96.1	123.0	149.9	176.9	203.8

Table 15. VBAR table showing International 1/4-inch bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 11).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	42.1	82.8	121.5	159.6	197.5	235.4			
12	38.3	79.0	117.6	155.8	193.7	231.5	269.3		
13	32.9	73.6	112.3	150.4	188.3	226.2	263.9	301.7	
14	26.3	67.0	105.7	143.8	181.8	219.6	257.4	295.1	
15	18.8	59.5	98.2	136.3	174.2	212.1	249.8	287.5	325.3
16		51.2	89.8	128.0	165.9	203.7	241.5	279.2	316.9
17		42.2	80.9	119.0	156.9	194.8	232.5	270.3	308.0
18		32.7	71.4	109.5	147.4	185.3	223.0	260.8	298.5
19		22.8	61.4	99.6	137.5	175.3	213.1	250.8	288.5
20		12.4	51.1	89.2	127.2	165.0	202.8	240.5	278.2
21			40.4	78.6	116.5	154.3	192.1	229.8	267.5
22			29.5	67.6	105.6	143.4	181.1	218.9	256.6
23			18.3	56.4	94.4	132.2	170.0	207.7	245.4
24			6.9	45.0	82.9	120.8	158.5	196.3	234.0
25				33.4	71.3	109.2	146.9	184.7	222.4

Table 16. VBAR table showing Scribner bd.ft./sq.ft. for various combinations of DBH and MH in sticks to a 9.6-in. top diameter limit (Equation 12).

DBH	MH in Sticks								
(Inches)	1	2	3	4	5	6	7	8	9
11	40.2	79.2	116.6	153.5	190.3	227.0			
12	36.8	75.8	113.1	150.1	186.9	223.6	260.2		
13	31.9	70.9	108.3	145.2	182.0	218.7	255.4	292.0	
14	26.0	65.0	102.3	139.3	176.1	212.8	249.4	286.0	
15	19.1	58.1	95.5	132.4	169.2	205.9	242.6	279.2	315.8
16		50.5	87.9	124.8	161.6	198.3	235.0	271.6	308.2
17		42.3	79.7	116.6	153.4	190.1	226.8	263.4	300.0
18		33.6	71.0	108.0	144.7	181.4	218.1	254.7	291.3
19		24.5	61.9	98.8	135.6	172.3	209.0	245.6	282.2
20		15.1	52.4	89.4	126.2	162.9	199.5	236.1	272.8
21			42.7	79.6	116.4	153.1	189.7	226.4	263.0
22			32.6	69.6	106.4	143.1	179.7	216.3	252.9
23			22.4	59.3	96.1	132.8	169.4	206.1	242.7
24			11.9	48.8	85.6	122.3	159.0	195.6	232.2
25				38.2	75.0	111.7	148.3	185.0	221.6

Table 17. Regression equations for predicting cubic-foot VBARs (CR) and Doyle (DR), International 1/4-inch (IR) and Scribner (SR) board-foot VBARs from the other three types of VBARs.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CR}=0.9234+0.2814DR-0.000499DR^2$	0.990	0.60
$\hat{CR}=0.7570+0.1755IR-0.000093IR^2$	0.994	0.48
$\hat{CR}=0.7684+0.1839SR-0.000118SR^2$	0.994	0.48
$\hat{DR}=0.5538+2.8328CR+0.053907CR^2$	0.990	2.80
$\hat{DR}=0.9852+0.5680IR+0.000845IR^2$	0.996	1.66
$\hat{DR}=0.8455+0.6021SR+0.000800SR^2$	0.997	1.45
$\hat{IR}=-2.4962+5.4058CR+0.027395CR^2$	0.994	2.95
$\hat{IR}= 0.2597+1.6456DR-0.002114DR^2$	0.997	2.18
$\hat{IR}=-0.01888+1.0511SR-0.000128SR^2$	1.000	0.27
$\hat{SR}=-2.1886+5.1045CR+0.031446CR^2$	0.994	2.90
$\hat{SR}= 0.1123+1.5740DR-0.001845DR^2$	0.997	1.87
$\hat{SR}= 0.03778+0.9506IR+0.000118IR^2$	1.000	0.27

more accuracy is needed and DBH is already measured for some other purpose(s), Equations 9-12 may be used.

If the user wants to predict one type of volume or VBAR from another, the appropriate equations in Tables 6 or 17 can be used, respectively. If the user needs to estimate board-foot cubic-foot ratios for one of the types of board-foot volumes, the appropriate equation in Table 7 can be used.

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