Individual Tree Volume Equations for Red Pine in Michigan

Gary W. Fowler, *School of Natural Resources and Environment, The University of Michigan, Ann Arbor, MI 48109-1115.*

ABSTRACT. *New total, pulpwood, sawtimber, and residual pulpwood cubic foot individual tree volume equations were developed for red pine in Michigan using nonlinear and multiple linear regression. Equations were also developed for Doyle, International 1/4 in., and Scribner bdft volume, and a procedure for estimating pulpwood and residual pulpwood rough cord volumes from the appropriate cubic foot equations was described. Average ratios of residual pulpwood (i.e., topwood, cubic foot or cords) to mbf were developed for 7.6 and 9.6 in. sawtimber. Data used to develop these equations were collected during May–August 1983–1985 from 3,507 felled and/or standing trees from 27 stands in Michigan. Sixteen and 11 stands were located in the Upper and Lower Peninsulas, respectively. All equations were validated on an independent data set. Rough cord volume estimates based on the new pulpwood equation were compared with contemporary tables for 2 small cruise data sets. The new equations can be used to more accurately estimate total volume and volume per acre when cruising red pine stands. North. J. Appl. For. 14(2):53–58.*

Composite and red pine individual tree volume equations have been developed for the Lake States by Gevorkiantz and Olsen (1955) and Buckman (1962), respectively. Ek and Droessler (1986) stated that Table 6 of Gevorkiantz and Olsen gives substantial overestimates of volumes of large trees and modified this table for volumes to a constant 4 in. top diameter limit. Buckman's equations are used in the "Managers Handbook for Red Pine in the North Central States" by Benzie (1977). The tables of Gevorkiantz and Olsen or some modifications of them are still widely used in Michigan for red pine by a variety of organizations including the Michigan Department of Natural Resources (MDNR) and the USDA Forest Service.

Since it has been 40 yr since Gevorkiantz and Olsen's study, and 33 yr since Buckman's study, it is time to reexamine individual tree volume equations for red pine. The MDNR has been concerned that the volume tables they are currently using appear to give inaccurate estimates of actual volumes cut at the mill. This is due, in part, to changing size and age distribution of red pine trees, increasing proportion of merchantable red pine plantations, changing of tree form, and/or decreasing of minimum top diameters. Better prediction models can now be developed using computerized statistical procedures such as nonlinear regression. The objective of this study was to develop new total, pulpwood, sawtimber, and residual pulpwood individual tree volume equations for red pine in Michigan.

Procedures

Felled and/or standing tree measurements were made during May–August 1983–1985 on a total of 3,507 trees from 27 red pine stands in Michigan [2,341 trees from 16 stands in the Upper Peninsula (U.P.), and 1,166 trees from 11 stands in the Lower Peninsula (L.P.)]. Five, four, and seven stands were selected from the eastern, central, and western U.P., respectively, and four, four, and three stands from the northeast, northwest, and southern L.P., respectively. Stands were selected from the above six regions to roughly represent the range of site index, age, stand density, average diameter at breast height (dbh), and average height found in Michigan.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 yr, basal area varied from 90 to 225 ft^2/ac, average dbh varied from 6.7 to 17.7 in., average total height (TH) varied from 33.2 to 86.0 ft, and average merchantable height (i.e., pulpwood, merchantable height) to an approximate 3.6 in. minimum top diameter (PH) varied from 2.0 to 8.4 100 in. sticks. Number of trees and average dbh, TH, and PH for the 27 stands are given in Table 1 for the prediction and validation data sets used in this study.

For felled trees, dbh to the nearest 0.1 in., TH to the nearest foot, PH to the nearest 100 in. stick to an approximate 3.6 in. minimum top diameter, and diameter inside

Table 1. Number of trees (NT) and average dbh in in., total height (TH) in ft, and pulpwood merchantable height (PH) in 100 in. sticks for the prediction and validation data sets based on 27 stands.

		Prediction				Validation			
Area	Stand	NT	Dbh	TH	PH	NT	Dbh	TH	PH
Upper Peninsula									
E	1	111	9.5	61.7	5.2	28	9.8	61.7	5.3
	2	115	7.3	50.1	3.4	30	6.9	49.8	3.1
	3	131	8.4	50.9	3.7	33	8.9	52.7	4.0
	4	127	10.2	52.1	4.4	33	9.9	53.1	4.5
	5	141	11.3	65.3	5.9	36	11.9	65.7	6.1
C	1	139	8.7	59.9	4.8	36	8.3	59.3	4.6
	2	99	7.8	53.0	4.1	26	7.5	52.9	4.0
	3	96	6.6	46.4	2.8	25	6.9	47.1	3.1
	4	174	15.9	60.1	4.8	45	16.1	58.8	4.8
W	1	92	9.1	68.3	5.9	23	9.5	68.4	5.9
	2	97	8.8	57.5	4.6	25	8.4	58.6	4.6
	3	102	9.4	65.3	5.5	26	9.2	64.7	5.5
	4	105	15.9	83.8	8.4	27	16.0	84.9	8.5
	5	114	17.5	84.1	8.3	29	18.7	82.8	8.1
	6	109	9.0	61.4	5.1	28	8.9	62.1	5.2
	7	111	8.2	50.1	3.7	28	8.3	50.1	3.8
Lower Peninsula									
NE	1	97	8.4	46.6	3.6	25	9.1	46.7	3.6
	2	112	10.3	73.0	6.6	29	9.6	71.6	6.2
	3	72	6.9	33.2	2.0	19	6.9	34.2	2.1
	4	81	14.4	85.9	8.6	21	14.5	86.0	8.6
NW	1	92	8.9	64.9	5.4	24	8.9	65.2	5.5
	2	97	12.5	72.7	6.9	25	12.7	72.7	6.9
	3	71	7.5	33.8	2.0	18	7.4	33.0	1.9
	4	89	8.3	51.3	4.1	23	9.2	51.8	4.4
S	1	71	7.9	49.0	4.0	19	7.7	48.6	3.9
	2	78	10.7	75.6	7.1	20	9.8	74.6	7.0
	3	66	6.6	46.7	3.2	17	7.2	47.7	3.4

(dib) and outside (dob) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft), dbh height (4.5 ft), several upper stem taper breaks, approximate 3.6 in. dib height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate dib's for standing trees (Fowler and Hussain 1987a, Fowler and Damschroder 1988).

PH to an approximate 3.6 in. minimum top diameter is defined as the number of 100 in. sticks that can be cut out of a tree with a minimum inside bark top diameter no smaller than 3.6 in. For standing trees, the minimum top diameter was decreased for trees where the last stick had a minimum top diameter of 3.6 in. at a length of at least 6 ft, and a full 100 in. stick could be cut from the tree. For felled trees, the last stick sometimes had a minimum top diameter less than 3.6 in.

Sawtimber trees were defined as trees that had at least one 100 in. stick (i.e., 8 ft log) with a minimum top diameter no smaller than 7.6 or 9.6 in. Sawtimber merchantable height is defined as the number of 100 in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 in. (SH8) or 9.6 in. (SH10). The MDNR uses 100 in. as the standard log length for both pulpwood and sawtimber because it yields easy, consistent merchantable height measurements and most logging operators cut 100 in. sticks to meet transportation and/or mill specifications. Thus, 100 in. sticks that are large enough in diameter and of adequate quality to be sawed into lumber are designated sawtimber. There were a total of 2,157 and 1,349 7.6 and 9.6 in. sawtimber trees, respectively.

For each tree, cubic foot volumes were calculated for each 100 in. stick using Smalian's formula. The volume of the butt stick was determined by dividing the stick into 2 pieces at dbh height, calculating the volume separately for each piece using Smalian's formula, and summing the 2 volumes. For each appropriate 100 in. stick, cubic foot and bd ft volumes were calculated using the following formulas:

Cubic foot: $V = \frac{(B+b)}{2} L$ [Avery and Burkhart 1994]

Doyle: $V = 0.5D^2 - 4.0D + 8.0$ [Husch et al. 1982]

Int. 1/4 in.: $V = 0.905(0.44D^2 - 1.20D - 0.30)$ [Husch et al. 1982]

Scribner: $V = 0.395D^2 - 0.99D - 2.15$ [Bruce and Schumacher 1950]

where

V = volume in cubic feet or board feet,

L = length of stick (100 in.) in ft (L = 8 ft for bd ft equations),

Table 2. Mean ($\bar{x}$) and minimum and maximum values of dbh, height, and cubic foot volume, and sample size *(n)* for the 4 data sets used to construct and validate the 4 types of volume prediction equations.

	Dbh (in.)			Height[a]			Ft^3 volume			
Prediction	$\bar{x}$	Min.	Max.	$\bar{x}$	Min.	Max.	$\bar{x}$	Min.	Max.	*n*
Total	10.1	3.6	23.9	60.0	23.3	100.8	19.15	1.08	105.05	3,507
Pulpwood	10.2	4.6	23.9	5.2	1	10	18.47	0.65	103.68	3,490
7.6 in. sawtimber	12.3	7.9	23.9	3.7	1	9	21.34	2.52	99.56	2,157
9.6 in. sawtimber	13.9	9.8	23.9	3.2	1	7	24.02	4.24	94.91	1,349

[a] Height is in ft for the total prediction equation and in 100 in. sticks for the other three prediction equations.

B = cross-sectional area inside bark of large end of the stick in ft^2,

b = cross-sectional area inside bark of small end of stick in ft^2, and

D = diameter of small end of the stick inside bark in in.

Pulpwood cubic foot volumes for each tree were determined by summing up the volumes of all sticks to an approximate 3.6 in. top diameter limit. Cubic foot and the 3 bd ft 7.6 and 9.6 in. sawtimber volumes for each tree were determined by summing up the volumes of all sawtimber sticks to approximate 7.6 and 9.6 in. top diameter limits, respectively. Total cubic foot volume for each tree was determined by adding the volume of the tree top (assumed to be a cone) to the pulpwood cubic foot volume. Residual pulpwood cubic foot volume above sawtimber cubic foot volume was determined as the difference between pulpwood and sawtimber cubic foot volume. Pulpwood volumes in rough cords were obtained by dividing cubic foot volumes by 77–84 ft^3/cord, depending on the average dob of all sticks for trees with pulpwood merchantable heights varying from 1–10 sticks (Taras 1956, Avery and Burkhart 1994), and multiplying the result by 0.96 to compensate for the extra 4 in. of stick beyond 8 ft. Cubic foot volumes were based on stick lengths of 100 in., while cordwood volume is based on a nominal cord, which assumes a stick length of 8 ft (i.e., 96 in.). Residual pulpwood volumes in rough cords were obtained by dividing cubic foot volume by 79 ft^3/cord and multiplying by 0.96.

Individual tree volume was regressed on various models of tree height and dbh using nonlinear and multiple linear regression. The mean and minimum and maximum values of dbh, total or merchantable height, and cubic foot volume and number of trees (*n*) for the four data sets used to construct and validate the four types of volume prediction equations are shown in Table 2. The mean and minimum and maximum values of Doyle, International 1/4 in., and Scribner bd ft volumes for the 2 sawtimber data sets used to construct and validate the 2 types of sawtimber volume prediction equations are shown in Table 3.

Results and Discussion

The data set used to develop the regression equations consisted of 2,789 trees from the 27 stands (Table 1), which is 79.5% of the number of trees in the study. This prediction data set was developed by randomly selecting approximately 80% of the trees from each stand.

The total, pulpwood, 7.6 in., and 9.6 in. sawtimber volume equations were based on 2,789, 2,774, 1,725, and 1,349 trees, respectively. The residual volume equations were based on 1,671 and 1,349 trees for 7.6 and 9.6 in. sawtimber, respectively (i.e., those sawtimber trees that had sawtimber volumes and residual pulpwood volumes above the sawtimber sticks). For 7.6 in. sawtimber the number of residual pulpwood sticks varied from 1-6, 1-5, 1-4, 1-4, 1-4, 1-3, 1-3, and 1-2 for 1, 2, 3, 4, 5, 6, 7, and 8 sawtimber sticks, respectively. One tree with 9 sawtimber sticks had 1 residual pulpwood stick. For 9.6 in. sawtimber, the number of residual pulpwood sticks was 2-8, 1-7, 2-6, 1-6, 1-5, 1-4, and 1-3 for 1, 2, 3, 4, 5, 6, and 7 sawtimber sticks, respectively.

Total, Pulpwood, and Sawtimber Prediction Equations

A comparison of various multiple linear and nonlinear regression equations based on goodness-of-fit (i.e., smallest standard error of the estimate $S_{y \cdot x}$ and largest coefficient of multiple determination R^2) and simplicity indicated that the best prediction equation for volume was:

$$\hat{V} = \hat{\beta}_0 D^{\hat{\beta}_1} H^{\hat{\beta}_2}$$

where $\hat{V}$ is predicted volume, D is dbh in in., and H = TH in ft or PH, SH8, or SH10 in 100 in. sticks. $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the sample regression coefficients related to the independent

Table 3. Mean ($\bar{x}$) and minimum and maximum values of Doyle, International 1/4 in., and Scribner bd ft volume for the 2 sawtimber data sets used to construct and validate the 2 types of sawtimber volume prediction equations.

	Type of sawtimber board-foot volume								
	Doyle			Int. ¼ in.			Scribner		
Minimum top diameter	$\bar{x}$	Min.	Max.	$\bar{x}$	Min.	Max.	$\bar{x}$	Min.	Max.
7.6 in.	85.8	5.0	554.2	124.2	12.4	643.5	119.6	11.1	633.8
9.6 in.	109.2	15.4	542.2	145.9	25.7	621.8	142.1	24.4	613.5

variables estimated using the Quasi-Newton nonlinear minimization method.

Table 4 shows the total, pulpwood, and 7.6 and 9.6 in. sawtimber prediction equations along with $S_{y \cdot x}$, R^2, and sample size n. Pulpwood rough cord volume can be estimated by dividing pulpwood cubic foot volume estimates by 77, 79, 80, 81, 82, 83, and 84 ft^3/cord for trees with 1, 2–3, 4, 5, 6, 7, and 8–10 sticks, respectively, and multiplying the result by 0.96.

Residual Pulpwood Prediction Equations

Two prediction equations were developed for the proportion of residual pulpwood volume in a sawtimber tree using multiple linear regression:

1. Height independent variables

$$\hat{P} = \hat{\beta}_0 + \hat{\beta}_1 \frac{RH}{PH} + \hat{\beta}_2 \frac{1}{RH} + \hat{\beta}_3 \frac{RH}{SH}$$

2. Height and dbh independent variables

$$\hat{P} = \hat{\beta}_0 + \hat{\beta}_1 \frac{RH}{PH} + \hat{\beta}_2 \frac{1}{RH} + \hat{\beta}_3 \frac{RH}{SH} + \hat{\beta}_4 \frac{1}{D}$$

where $\hat{P}$ is the predicted proportion of residual pulpwood cubic foot volume, SH is the number of sawtimber sticks, and RH is the residual number of pulpwood sticks above sawtimber sticks (i.e., $RH = PH - SH$). Table 5 shows the two prediction equations for 7.6 and 9.6 in. sawtimber along with $S_{y \cdot x}$, R^2, and n.

Predicted residual pulpwood volume can be obtained by multiplying $\hat{P}$ times predicted pulpwood cubic foot volume (Table 4). Predicted residual pulpwood rough cord volume can be obtained by dividing residual pulpwood cubic foot volume by 79 ft^3/cord and multiplying the result by 0.96. For sawtimber trees with residual pulpwood volume, sawtimber cubic foot volume can be estimated by subtracting predicted residual pulpwood cubic foot volume from predicted pulpwood volume.

To simplify field procedures in cruising, sometimes only the number of sawtimber sticks is measured, and the residual pulpwood volume is estimated using a ratio of residual pulpwood volume in cubic feet or rough cords to sawtimber volume in mbf. The ratio of average cubic-foot and rough cord volume to average mbf volume was developed for 7.6 and 9.6 in. sawtimber based on n = 2157 and 1349 trees, respectively. Rough cord volume was estimated from cubic foot volume by dividing cubic foot volume by 79 and multiplying the result by 0.96. Table 6 shows these ratios for the three types of bd ft volumes for 7.6 and 9.6 in. sawtimber.

Validation

The data set used to validate the prediction equations consisted of 718 trees from the 27 stands, which is 20.5% of the number of trees in the study. Number of trees and average dbh, TH, and PH for the 27 stands are given in Table 1. The total, pulpwood, and 7.6 in. sawtimber and residual pulpwood equations were validated on 718, 716, and 432 trees, respectively. Residual pulpwood volume was estimated by multiplying the predicted proportion of residual pulpwood based on height independent variables times the predicted pulpwood cubic foot volume.

For each volume equation, the average relative error as a percent ($\overline{RE}$) was calculated where

$$\overline{RE} = \sum_{i=1}^{n} RE_i / n$$

and $RE_i = [(\hat{V}_i - V_i)/V_i] \times 100$, $\hat{V}_i$ and V_i are the predicted and actual volumes for the ith tree, and n is the number of trees in the validation data set. The relative error as a percent for the sum of the predicted volumes was also calculated where

$$RE_S = \left[\left(\sum_{i=1}^{n} \hat{V}_i - \sum_{i=1}^{n} V_i \right) / \sum_{i=1}^{n} V_i \right] \times 100$$

Table 4. Estimated parameters ($\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$), standard errors of the estimate *($S_{y \cdot x}$)*, coefficients of determination *(R^2)*, and sample sizes *(n)* for the total, pulpwood, and 7.6 and 9.6 in. sawtimber volume prediction equations.

Prediction equation[a]	Units	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$s_{Y \cdot X}$	R^2	n
Total	Ft^3	0.002979	1.7143	1.1287	2.02	0.986	2,789
Pulpwood	Ft^3	0.07000	1.7349	0.8348	1.62	0.991	2,774
7.6 in. sawtimber	Ft^3	0.1035	1.614	0.8688	1.64	0.991	
	Bd ft Doyle	0.06032	2.280	0.9635	13.50	0.978	1,725
	Bd ft Int. ¼ in.	0.3665	1.753	0.9497	15.59	0.984	
	Bd ft Scribner	0.3034	1.807	0.9542	14.16	0.983	
9.6 in. sawtimber	Ft^3	0.1367	1.526	0.8878	1.52	0.993	
	Bd ft Doyle	0.1404	2.020	0.9714	12.72	0.983	1,349[b]
	Bd ft Int. ¼ in.	0.6381	1.582	0.9652	13.09	0.987	
	Bd ft Scribner	0.5594	1.619	0.9669	13.08	0.987	

a $\hat{V} = \hat{\beta}_0 D^{\hat{\beta}_1} H^{\hat{\beta}_2}$

b The 9.6 in. sawtimber equations were not validated, so all 9.6 in. sawtimber trees were used to develop the prediction models.

Table 5. Estimated parameters $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$, and $\hat{\beta}_4$, $S_{y.x}$, R^2, and n for the proportion of residual pulpwood cubic foot volume prediction equations for 7.6 and 9.6 in. sawtimber.

Prediction equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$s_{Y\cdot X}$	R^2	n
7.6 in. sawtimber								
Height[a]	–0.1943	0.9444	0.1010	0.02757	—	0.0215	0.990	1,671
Height & dbh[b]	–0.2384	0.7804	0.08480	0.03580	1.2740	0.0187	0.992	1,671
9.6 in. sawtimber								
Height[a]	–0.2969	1.0567	0.2076	0.02102	—	0.0314	0.980	1,349
Height & dbh[b]	–0.3696	0.7407	0.1397	0.02952	3.3576	0.0234	0.989	1,349

a $\hat{P} = \hat{\beta}_0 + \hat{\beta}_1 \frac{RH}{PH} + \hat{\beta}_2 \frac{1}{RH} + \hat{\beta}_3 \frac{RH}{SH}$

b $\hat{P} = \hat{\beta}_0 + \hat{\beta}_1 \frac{RH}{PH} + \hat{\beta}_2 \frac{1}{RH} + \hat{\beta}_3 \frac{RH}{SH} + \hat{\beta}_4 \frac{1}{D}$

and

$$\sum_{i=1}^{n} \hat{V}_i \text{ and } \sum_{i=1}^{n} V_i$$

are the sum of the predicted and actual volumes, respectively.

$\overline{RE}$, the minimum and maximum values of RE_i, and RE_S for the total, pulpwood, and 7.6 in. sawtimber prediction equations are shown in Table 7. While there is a wide range of relative error in predicting the volumes for an individual tree, $\overline{RE}$ and RE_S indicate that for a large number of trees the prediction equations yield valid estimates on the average.

One sample was taken in each of two red pine stands using 0.05 ac circular plots. For each stand, dbh to the nearest 0.1 in. and pulpwood merchantable height to the nearest 100 in. stick to an approximate 3.6 in. minimum top diameter was measured for each tree with dbh ≥ 4.6 in. on each plot.

Four plots were selected from Stand 1 in the Lower Peninsula. Dbh varied from 6.0 to 10.4 in., PH varied from 2 to 4 sticks, and the number of trees per plot varied from 20 to 30. Five plots were selected from Stand 2 in the Upper Peninsula. Dbh varied from 4.9 to 20.7 in., PH varied from 1 to 10 sticks, and number of trees per plot varied from 8 to 14.

Volume in rough cords was estimated for both stands using the new cubic foot pulpwood prediction equation and converting to rough cords, Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Results for the two samples are shown in Table 8. For Stand 1, the new equation yielded estimates 13.5 and 25.8% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively. For Stand 2, the new equation yielded estimates 2.7% lower and 12.4% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively. Rough cord volumes for 9-stick trees were obtained by adding 0.06 cords to rough cord volumes for 8-stick trees in Table 6 of Gevorkiantz and Olsen. For one 19 in. tree, rough cord volumes for 10 sticks was obtained by adding 0.05 cords to rough cord volume for 9 sticks.

The new equation yielded estimates that were higher than those of Gevorkiantz and Olsen for smaller trees (Stand 1) and somewhat lower for larger trees (Stand 2). The new equation yielded estimates that were higher than those of Ek and Droessler with the difference being larger for smaller trees. The estimates of Ek and Droessler were less than those of Gevorkiantz and Olsen with the difference being larger for larger trees. In comparing these results, note that the new equation used cubic foot rough cord values varying from 77 to 84 while Gevorkiantz and Olsen and Ek and Droessler used 79. Also, the new equation was based on an approximate 3.6 in. minimum top diameter while Gevorkiantz and Olsen used variable top diameters that were, in general, not less than 4 in. except for some smaller trees that had top diameters between 3 and 4 in.; Ek and Droessler used a 4 in. top diameter.

For more detail on the development of the volume prediction equations in this study, see Fowler and Hussain (1987b, 1989a, 1989b).

Table 6. Ratio of residual pulpwood volume in cubic feet and rough cords to sawtimber volume in mbf for Doyle, International 1/4 in., and Scribner bd ft volume for 7.6 and 9.6 in. sawtimber.

	Minimum top diameter			
	7.6 in.		9.6 in.	
Type of sawtimber volume	Ft³/mbf	Cords/mbf	Ft³/mbf	Cords/mbf
Doyle	59.15	0.7188	93.21	1.1327
International ¼ in.	40.84	0.4963	69.72	0.8472
Scribner	42.40	0.5152	71.59	0.8700

Table 7. $\overline{RE}$, minimum and maximum RE_i, values, RE_S, and the number of trees in the validation data set for the total, pulpwood, and 7.6 in. sawtimber prediction equations.

Prediction equation[a]	Relative error			RE_S[a]	n
	$\overline{RE}$	Min.	Max.		
Total ft^3	3.2	−21.3	54.8	0.45	718
Pulpwood ft^3	2.8	−16.6	26.1	0.72	716
7.6 in. sawtimber					
Ft^3	1.2	−19.2	16.2	1.1	
Residual pulpwood ft^3	−1.3	−72.8	127.6	−0.61	
Doyle	4.3	−32.1	41.0	2.4	432
Int. 1/4 in.	1.1	−25.7	24.8	1.4	
Scribner	1.5	−26.3	26.7	1.4	

[a] The relative error as a percent for the sum of the predicted volumes. See text for the formula.

Comments

The new volume prediction equations can be used to estimate pulpwood, sawtimber, or sawtimber and residual pulpwood volume when cruising red pine stands. The equations can be used to develop volume tables or entered into a computer program to facilitate computer volume calculations for cruise data.

Table 8. Volume estimates in rough cords per acre for the samples taken from 2 stands, one in the Lower Peninsula and one in the Upper Peninsula, based on the new pulpwood cubic foot volume prediction equation, Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Stand	New equation	Gevorkiantz and Olsen (Table 6)	Ek and Droessler (Table 1)
1	42.0	37.0	33.4
2	87.7	90.2	78.1

Literature Cited

AVERY, T.E., AND H.E. BURKHART. 1994. Forest measurements. McGraw-Hill, New York. 408 p.

BENZIE, J.W. 1977. Manager's handbook for red pine in the North Central States. USDA For. Serv. Gen. Tech. Rep. NC-33. 22 p.

BRUCE, D., AND F. SCHUMACHER. 1950. Forest mensuration. McGraw-Hill, New York. 483 p.

BUCKMAN, R.E. 1962. Growth and yield of red pine in Minnesota. USDA For. Serv. Tech. Bull. 1272. 50 p.

EK, A.R., AND T.D. DROESSLER. 1986. An error in the application of the composite volume table: Table 6 of USDA Tech. Bull. 1104. North. J. Appl. For. 3:88.

FOWLER, G.W., AND L.J. DAMSCHRODER. 1988. A red pine bark factor equation for Michigan. North. J. Appl. For. 5:28–30.

FOWLER, G.W., AND N.G. HUSSAIN. 1987a. Bark factor equation for red pine. Mich. Dep. Natur. Resour. For. Inf. Leaflet 1-87. 2 p.

FOWLER, G.W., AND N.G. HUSSAIN. 1987b. Individual tree volume equations for red pine in Michigan. Mich. Dep. Natur. Resour. 23 p.

FOWLER, G.W., AND N.G. HUSSAIN. 1989a. Individual tree sawtimber volume equations for red pine in Michigan. Mich. Dep. Natur. Resour. 21 p.

FOWLER, G.W., AND N.G. HUSSAIN. 1989b. Sawtimber volume and volume-basal area ratio equations for red pine in Michigan: 9.6-in. minimum top diameter. Mich. Dep. Natur. Resour. 31 p.

GEVORKIANTZ, S.R., AND L.P. OLSEN. 1955. Composite volume tables for timber and their application in the Lake States. USDA Tech. Bull. 1104. 51 p.

HUSCH, B., C.I. MILLER, AND T.W. BEERS. 1982. Forest mensuration. Wiley, New York. 402 p.

TARAS, M.A. 1956. Buying pulpwood by weight as compared to volume measure. USDA For. Serv. Res. Pap. SE-74. 11 p.