A Red Pine Bark Factor Equation for Michigan

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ABSTRACT. A multiple linear regression equation was developed to predict bark factor for red pine in Michigan as a function of tree height. The equation was validated on independent data sets. The prediction equation yielded average relative errors less than ±1% at all tree heights. Procedures are described for using the bark factor equation to estimate diameter inside bark from diameter outside bark and vice versa at any tree height. Specific uses of the prediction equation are also discussed.

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Bark factor (F) at a given tree height is the ratio of diameter inside bark to diameter outside bark. Bark factors vary with species, age, site, and tree height. Bark factors at stump or breast height usually vary from 0.87 to 0.93. Even though much of the variation in bark factor is related to species, bark factor does increase with tree height for many species. In spite of this relationship, a constant bark factor has been assumed for many species for all tree heights. The use of a constant bark factor, determined at breast height, for all tree heights will, in general, lead to underestimates of most tree and log volumes and overestimates of bark volume.

Multiple linear regression equations have been developed to predict bark factor as a function of various independent variables such as tree height and associated diameter outside bark. Such equations have not been developed for many species because data has been lacking for the independent variables or the use of a constant bark factor has been considered adequate. As forest management becomes more intensive, the use of such equations should be considered. See Husch et al. (1982) for a detailed discussion on bark factors.

Bark factors can be used to estimate (1) peeled log or stem volume from unpeeled log or stem volume, (2) bark residue associated with the manufacturing process, (3) the quantity of bark available where there is a demand for bark, (4) past diameter and diameter growth from increment cores, and (5) inside bark diameters from outside bark diameter measurements at various heights on standing trees to facilitate volume calculations.

The objective of this study was to develop a bark factor prediction equation for red pine in Michigan.

PROCEDURES

Felled tree measurements were made on a total of 576 trees from 6 stands (2 each in the western, central, and eastern U.P.) and 279 trees from 4 stands (2 each in the southern and northern L.P.) in Michigan's Upper (U.P.) and Lower Peninsula (L.P.), respectively. Diameter inside and outside bark were measured to the nearest 0.1-in. at stump height and at the top of each 8.3-ft bolt (100-in. stick) cut out of each tree to an approximate 3.6-in. diameter top limit. The number of trees and average and range of dbh in inches and merchantable height in 8.3-ft bolts are shown in Table 1. This prediction data set was supplemented by bark factor measurements at 4.5 ft above the ground (i.e., dbh to the nearest 0.1-in. and bark thickness to the nearest 0.05-in. with a bark punch) from a sample of 100 trees (20 from each of 5 stands) in the L.P.

The data set used to validate the prediction equation consisted of a random sample of 20 trees from each of 3 stands not used to construct the prediction equation (2 from the U.P. and 1 from the L.P.). The number of trees and average and range of dbh in inches and merchantable height in feet are shown in Table 2. Merchantable heights are given in feet because variable bolt lengths were cut from trees from 2 of these stands. This validation data set was supplemented by bark factor measurements at 4.5 ft above the ground from a sample of 50 trees (10 from each of 5 stands) in the L.P.

For the prediction data set, the bark factor at each tree height was determined using all of the trees with measurements at that height with the formula

$$F = \frac{\text{sum of diameters inside bark}}{\text{sum of diameters outside bark}}$$

A good discussion on equations to determine bark factor is presented in Husch et al. (1982).

Table 1. Number of trees, average ($\bar{x}$) and range (R) of *DBH* in inches and merchantable height (M. Ht.) in 100 in. sticks for the 10 data sets used to construct the prediction equation.

Area		Stand	No. of trees	DBH $\bar{x}$	DBH R	M. Ht. $\bar{x}$	M. Ht. R
U.P.	E	1	102	7.2	3.6–11.2	3.7	1–5
		2	64	8.5	4.0–15.0	2.9	1–5
	C	1	140	8.6	4.0–13.4	4.8	2–6
		2	96	6.6	3.6–9.8	3.8	2–5
	W	1	42	17.8	11.0–23.6	7.3	5–9
		2	132	8.3	4.5–12.7	4.5	1–7
L.P.	N	1	82	7.5	5.2–9.6	3.2	1–4
		2	83	8.2	5.2–10.6	5.1	3–7
	S	1	62	7.6	4.7–10.8	4.7	1–7
		2	52	7.2	4.8–9.9	3.2	1–5

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Table 2. Number of trees, average ($\bar{x}$) and range (*R*) of DBH in inches and merchantable height (M. Ht.) in feet for the 3 data sets used to validate the prediction equation.

Area	Stand	No. of trees	DBH $\bar{x}$	DBH *R*	M. Ht. $\bar{x}$	M. Ht. *R*
U.P.	1	20	7.7	5.2–9.5	36.8	32–40
	2	20	15.4	10.9–20.9	37.5	24–56
L.P.	1	20	9.9	5.8–13.1	53.1	36–64

Note: Top limits for Stand 2—U.P. varied from 5–10 in. Bolt lengths for stands 2 (U.P.) and 1 (L.P.) varied from 8–20 ft and 6–8 ft, respectively. Bolt length for Stand 1 was 8 ft. A 1-in. trim allowance is assumed for each 2 ft of bolt length.

Table 3. Number of trees, pooled average bark factors, and predicted bark factors from the prediction equation for 11 tree heights in feet.

Tree height	No. of trees	Bark factor	Predicted bark factor
0.5	855	0.901	0.901
4.5	100	0.940	0.952
8.8	855	0.960	0.960
17.1	836	0.970	0.968
25.4	768	0.974	0.972
33.7	620	0.975	0.975
42.0	351	0.976	0.977
50.3	174	0.975	0.979
58.6	41	0.970	0.981
66.9	18	0.972	0.982
75.2	1	0.978	0.983

RESULTS AND DISCUSSION

The variation of average F at a given height among stands and regions (i.e., western, central, and eastern U.P., and southern and northern L.P.) and between the U.P. and L.P. was relatively small, justifying the pooling of all of the data for a given height (Table 3). For each height, average F's for the 5 regions varied by no more than 0.006 except for 0.5 ft where they varied by 0.039 (four regions varied by no more than 0.017). For each height, average F for the U.P. and L.P. varied by no more than 0.006. F was 0.901, 0.940, and 0.960 at tree heights of 0.5, 4.5, and 8.8 ft, respectively. For all other heights the F's ranged from 0.970 to 0.978.

Bark factor was plotted against tree height for the pooled data, indicating that bark factor (Y) would be very closely predicted by some combination of the following forms of tree height (X): X, $1/X$, and $\ln X$. A set of prediction equations (i.e., all combinations of X, $1/X$ and $\ln X$) was constructed using weighted multiple linear regression with weights based on the number of trees with measurements at that height for 11 heights (Table 3). The best prediction equation, i.e., that equation that yielded the smallest standard error of the estimate ($s_{y \cdot x}$) and the largest coefficient of multiple determination (R^2), was

$$\hat{Y} = 0.9405 - 0.01637 \left(\frac{1}{X}\right) + 0.009954 \ln X$$

$$R^2 = 0.99, \; s_{y \cdot x} = 0.0625$$

where $\hat{Y}$ is estimated F and X is tree height in feet. This regression equation was highly significant ($P < 0.001$). Predicted F's from the prediction equation for the 11 tree heights are shown in Table 3.

The prediction equation was validated on the 3 independent data sets (Table 2) for the following height classes in feet: 0.5, 6–9, 10–18, 19–26, 27–34, 35–43, 44–50, 51–59, and 60–67. Average relative errors as percentages ($\overline{RE}$) were calculated for each tree height class for each sample of 20 trees using the formula

$$\overline{RE} = \sum_{i=1}^{n} RE_i/n$$

where

$$RE_i = \frac{\hat{Y}_i - Y_i}{Y_i}(100),$$

$\hat{Y}_i$ = predicted F for the ith tree,
Y_i = observed F for the ith tree, and
n = number of trees with measurements in the specified height class.

For the sample from each of the 3 stands, $\overline{RE}$ was less than $\pm 1\%$ for each height class. Table 4 shows $\overline{RE}$, the range of relative errors, and the number of trees that had heights in each height class for the 3 stands. The range was generally between $\pm 4\%$. Only 3 trees had relative errors greater than $\pm 5\%$ (Table 4; U.P.—Stand 2 and L.P.—Stand 1). The prediction equation tends to somewhat overestimate F for larger tree heights.

The prediction equation was also used to estimate F at 4.5 ft for each of the sample of 10 trees from each of 5 stands. The average relative error for all 50 trees pooled was 0.91%. The range of relative errors for the 5 stands was −1.9 to 0.80, −0.14 to 1.6, −1.1 to 2.7, 0.62–3.5, and −1.3 to 3.7%, respectively.

Confidence intervals can be calculated for the true F for any desired value of tree height. Prediction intervals for an individual future prediction of F at a given height for a specific tree can not be directly calculated because the average F at each height was based on unequal sample sizes, necessitating the use of weighted multiple linear regression. The confidence intervals could be used to indirectly evaluate the accuracy of predictions. However, the validation results give a more direct evaluation of prediction accuracy. For discussions of weighted multiple linear regression, see Brownlee (1965), Draper and Smith (1981), and Steel and Torrie (1960). In evaluating the accuracy of the prediction equation, keep in mind that sample sizes decrease greatly as tree height increases (Table 3).

APPLICATIONS

The prediction equation can be used to estimate F at any tree height. Since $F = DIB/DOB$, DIB can be estimated

Table 4. Average relative errors ($\overline{RE}$), range of relative errors (Range), and number of trees for each height class in feet (*n*) for the 3 stands (validation data sets). All *RE* values are percentages.

Ht. class	U.P.—Stand 1 $\overline{RE}$	Range	*n*	U.P.—Stand 2 $\overline{RE}$	Range	*n*	L.P.—Stand 1 $\overline{RE}$	Range	*n*
0.5	−0.43	−3.7,3.3	20	0.30	−3.5,5.1	20	0.71	−2.8,6.5	20
6–9	−0.36	−1.4,1.0	20	1.20	—	1	1.12	−0.84,2.8	20
10–18	−0.27	−1.5,2.3	20	0.95	−1.2,4.8	19	0.37	−1.4,2.0	20
19–26	−0.06	−0.9,1.2	20	0.94	−0.87,8.4	12	0.22	−1.4,1.7	28
27–34	0.07	−1.0,1.5	20	0.94	−0.65,4.0	14	0.31	−1.2,1.8	20
35–43	0.37	−1.1,1.5	12	0.75	−1.2,3.1	14	0.50	−1.2,3.2	22
44–50				0.60	−0.77,3.2	8	0.40	−0.62,2.3	26
51–59				0.12	—	1	0.38	−0.55,3.5	17
60–67							0.83	0.77,3.8	8

as $\hat{DIB} = \hat{F} \cdot DOB$, and DOB can be estimated as $\hat{DOB} = DIB/\hat{F}$. Specific uses of the prediction equation include:

1. Estimating *DIB* from *DOB* measurements at various heights of standing trees to facilitate estimation of tree solid wood and bark volume. The tree can be divided into a number of stem sections, volumes estimated for each section assuming some appropriate geometrical solid, and the volumes of all sections summed to yield an estimate of tree volume.
2. Estimating *DIB* from *DOB* measurements on felled stem sections or logs to facilitate estimation of bark volume, or peeled volume from unpeeled volume. Husch et al. (1982) show that bark volume (V_B) is

$$V_B = V(1 - \hat{F}^2)$$

where V is the unpeeled volume of a log based on Huber's formula and $\hat{F}$ is the estimated bark factor at the midpoint of the log. The percent of the unpeeled log volume that is bark volume is given by $(1 - \hat{F}^2) \cdot 100$.
3. Past *DOB* and *DOB* growth can be determined from past *DIB* growth as follows:

$$\text{Past } DOB \text{ growth} = \text{Past } DIB \text{ growth}/\hat{F}$$

and

$$\text{Past } DOB = \text{Present } DOB - \text{Past } DOB \text{ Growth},$$

where past *DIB* growth might be obtained using an increment borer.
4. Estimating *DIB* at the top of the butt log to facilitate estimation of tree form (e.g., Girard form class). □

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