INDIVIDUAL TREE VOLUME EQUATIONS FOR RED PINE IN MICHIGAN

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Management Summary

New total, pulpwood, sawtimber, and residual pulpwood cubic-foot individual tree volume equations were developed for red pine in Michigan. Data used to develop these equations were collected from 27 red pine stands in Michigan (16 and 11 stands from the Upper and Lower Peninsulas, respectively).

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y \cdot x}$), and independent validation data sets for a series of linear and nonlinear regression equations indicated that: (1) nonlinear equations were most accurate for total, pulpwood, and sawtimber cubic-foot volume, and (2) linear equations were most accurate for residual pulpwood cubic-foot volume. The residual pulpwood volume equations using diameter at breast height (DBH) and height independent variables yielded somewhat higher R^2's and lower $s_{y \cdot x}$'s, and, in general, less accuracy than the equations using height independent variables. However, the differences between the 2 equations is relatively small, indicating that the use of the simpler height equation is adequate for most cruising situations. The new total volume equation will usually yield total cubic-foot volume per acre estimates from 8-15% lower to 5-10% higher than estimates based on Table 3 in Gevorkiantz and Olsen (1955). The new pulpwood volume equation will usually yield rough cord volume per acre estimates from 10-15% lower to 10-15% higher and from 10 to 25% higher than estimates based on Table 6 in Gevorkiantz and Olsen and Table 1 in Ek and Droessler (1986), respectively, depending on stand DBH.

We recommend the use of the following individual tree volume equations in most cruising situations for red pine:

1. Total volume

   $\hat{V}_T = 0.002974 D^{1.7143} H^{1.1287}$

2. Pulpwood volume

   $\hat{V}_P = 0.07000 D^{1.7349} H^{0.8348}$

3. Sawtimber volume

$$\hat{V}_S = 0.1087D^{1.5880}{}_H{}^{0.8804}$$

4. Residual pulpwood volume

$$\hat{V}_{RP} = \hat{P} \cdot \hat{V}_P, \text{ where}$$

$$\hat{P} = -0.1943 + 0.9444 \cdot \frac{RH}{PH} + 0.1010 \cdot \frac{1}{RH} + 0.02757 \cdot \frac{RH}{SH}$$

In the above equations, TH is total height in feet, PH is pulpwood merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit, SH is sawtimber merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit, RH is the residual number of pulpwood sticks above and beyond sawtimber sticks, and $\hat{P}$ is the predicted proportion of residual pulpwood volume. For trees with both sawtimber and residual pulpwood volume, we recommend that sawtimber volume be determined using $\hat{V}_S = \hat{V}_P - \hat{V}_{RP}$. For trees with just sawtimber, Equation 3 above should be used. Pulpwood and residual pulpwood rough cord volumes can be determined from the respective cubic foot volumes using appropriate cu.ft./cd. conversion values.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - INDIVIDUAL TREE VOLUME EQUATIONS

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Background

Composite and red pine individual tree volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955) and Buckman (1962), respectively. Ek and Droessler (1986) stated that Table 6 of Gevorkiantz and Olsen gives substantial overestimates of volumes of large trees and modified this table for volumes to a constant 4-in. top diameter limit. Buckman's equations are used in the "Managers Handbook for Red Pine in the North Central States" by Benzie (1977). The tables of Gevorkiantz and Olsen and Buckman or some modifications of them are still widely used in Michigan for red pine.

Purpose

The purpose of this paper is to present new total, pulpwood, sawtimber, and residual pulpwood cubic-foot volume equations and tables for red pine in Michigan. Rough cord volume equations and tables are also given for pulpwood and residual pulpwood.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 3507 trees from 27 stands as follows:

1) 2341 trees from 16 stands in the Upper Peninsula (i.e., 5, 4, and 7 stands each in the eastern, central, and western U.P., respectively), and
2) 1166 trees from 11 stands in the Lower Peninsula (i.e., 4, 4, and 3 stands from the northeast, northwest, and southern L.P., respectively).

Stands were selected from the above six regions to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1983-1985.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 years, basal area/acre varied from 90 to 225 sq.ft., average DBH varied from 6.7 to 17.7 in., average total height varied from 33.2 to 86.0 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 2.0 to 8.4 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987).

Merchantable height to an approximate 3.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum inside bark top diameter no smaller than 3.6 inches. This minimum top diameter was decreased for trees where the last stick had a minimum top diameter of 3.6-in. at a length of at least 6 ft. and a full 100-in. stick could be cut from the tree. For felled trees, the last stick sometimes had a minimum top diameter less than 3.6 inches. Merchantable height to an approximate 7.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. Pulpwood and sawtimber volumes were determined by summing up the volumes of sticks to approximate 3.6-in. and 7.6-in. top diameter limits, respectively. Total tree volume was determined by adding the volume of the tree top (assumed to be a cone) to the pulpwood volume. Residual pulpwood volume above and beyond sawtimber volume was determined as the difference between pulpwood and sawtimber volumes. Pulpwood volumes in rough cords were obtained by dividing cubic-foot volumes by 77-84 cu.ft./cd., depending on the average DOB of all sticks for trees with merchantable heights varying from 1-10 sticks (Taras 1956, Avery and Burkhart 1983), and multiplying the result by 0.96 to compensate for the extra 4 in. of stick length beyond 8 feet. Residual pulpwood volumes in rough cords were obtained by dividing cubic-foot volumes by 79 cu.ft./cd. and multiplying by 0.96.

Individual tree volume was regressed on various forms of tree height and DBH using multiple linear regression and nonlinear regression.

Results

The data set used to develop the regression equations consisted of approximately 80% of the trees in each of the 27 stands, yielding a total of 2789 trees.

The total volume equations were based on 2789 trees with DBH ranging from 3.6 to 23.9 in., total height ranging from 23.3 to 100.8 ft., and total cubic-foot volume ranging from 1.08 to 105.05 cu.ft.

The pulpwood volume equations were based on 2774 trees with DBH ranging from 4.6 to 23.9 in., merchantable pulpwood height varying from 1-10 sticks, and

pulpwood cubic-foot volume ranging from 0.65 to 103.68 cu.ft. A pulpwood tree had to have a DBH $\geq$4.6 in. with at least one 100-in. stick having an approximate minimum top diameter of 3.6 inches.

The sawtimber volume equations were based on 1742 trees with DBH ranging from 7.9 to 23.9 in., sawtimber merchantable height ranging from 1-9 sticks, and sawtimber cubic-foot volume ranging from 2.52 to 100.95 cu.ft. A sawtimber tree had to have at least one 100-in. stick with a minimum top diameter $\geq$7.6 inches.

The residual volume equations were based on 1671 trees (i.e., those sawtimber trees that had sawtimber volumes and pulpwood volumes). The number of residual pulpwood sticks varied from 1-6, 1-5, 1-4, 1-4, 1-4, 1-3, 1-3, and 1-2 for 1, 2, 3, 4, 5, 6, 7, and 8 sawtimber sticks, respectively. One tree with 9 sawtimber sticks had one residual pulpwood stick.

Total, pulpwood, and sawtimber volume prediction equations

Individual tree total volume equations of the form $V=\beta_o+\beta_1 D^2H$ were developed separately for each of the 27 stands. There were no significant differences between the stands in the U.P. and the stands in the L.P. using the one-way analysis of variance for intercepts (β_o) and the regression coefficients (β_1) with a level of significance $\alpha=0.05$. Therefore, all of the data from the 27 stands were pooled for each type of volume equation to develop one pooled prediction equation for Michigan.

A comparison of various multiple linear regression and nonlinear regression equations based on goodness-of-fit and simplicity indicated that the following nonlinear prediction equation compared favorably to all other equations examined for total, pulpwood, and sawtimber volumes:

$$\hat{V} = \hat{\beta}_o D^{\hat{\beta}_1} H^{\hat{\beta}_2}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is total height in ft. (TH), merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit (PH), or merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit (SH) for total, pulpwood, and sawtimber volumes, respectively. $\hat{\beta}_o$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the sample regression coefficients related to the independent variables.

Table 1 shows the total, pulpwood, and sawtimber volume prediction equations along with sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$),

Table 1. Estimated parameters ($\hat{\beta}_o$, $\hat{\beta}_1$, and $\hat{\beta}_2$), sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of determination (R^2) for the total, pulpwood, and sawtimber cubic-foot volume prediction equations.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	n	$s_{y \cdot x}$	R^2
(1) Total[a]	0.002974	1.7143	1.1287	2789	2.02	0.986
(2) Pulpwood[b]	0.07000	1.7349	0.8348	2774	1.62	0.991
(3) Sawtimber[c]	0.1087	1.5880	0.8804	1742	1.48	0.993

[a] $\hat{V} = 0.002974D^{1.7143}H^{1.1287}$.
[b] $\hat{V} = 0.07000D^{1.7349}H^{0.8348}$.
[c] $\hat{V} = 0.1087D^{1.5880}H^{0.8804}$.

and coefficients of determination (R^2). Total, pulpwood, and sawtimber volume tables are shown in Table 2, 3, and 4, respectively, in the Appendix. A pulpwood rough cord volume table is given in Table 5 in the Appendix. The values in this table were obtained by dividing the pulpwood volume values in Table 3 by 77, 79, 80, 81, 82, 83, and 84 cu.ft./cd. for trees with 1, 2-3, 4, 5, 6, 7, and 8-10 sticks, respectively, and multiplying the result by 0.96.

The total volume values in Table 2 are greater and less than the values in Table 3 of Gevorkiantz and Olsen (1955) for DBH's less than 14 in. and greater

than 19 in., respectively, with mixed results for DBH's between 14 and 19 inches.

The rough cord values in Table 5 are greater and less than the values in Table 6 of Gevorkiantz and Olsen for DBH's less than 9 in. and greater than 14 in., respectively, with mixed results for DBH's between 9 and 14 inches. The values in Table 5 are greater than the values in Table 1 of Ek and Droessler (1986) except for D=25 to 30 in. with PH=9, and D=29 and 30 in. with PH=8. In general, the values of Table 5 are between the values of Gevorkiantz and Olsen and those of Ek and Droessler for DBH's greater than 14 inches.

The tables of Gevorkiantz and Olsen and Ek and Droessler used a cubic-foot per rough cord value of 79 while we used values ranging from 77 for 1 stick trees to 84 for trees with 8 or more sticks. If we used the value 79, our values in Table 5 would be larger except for trees with 1-3 sticks.

Residual pulpwood volume prediction equations

Residual pulpwood volume in a sawtimber tree was estimated using two methods: 1) Use of a prediction equation for residual volume; and 2) multiplication of the estimated proportion of residual pulpwood volume in a sawtimber tree by the estimated pulpwood volume in the tree.

Method 1 -- Two prediction equations were developed for residual pulpwood cubic-foot volume:

1. Height independent variables

(4) $$\hat{V}=0.6776+1.7913\cdot RH-0.03273\cdot\frac{PH}{SH}$$

$R^2=0.934$, $s_{y\cdot x}=0.4809$, n=1671

2. Height and diameter independent variables

(5) $$\hat{V}=0.4346+1.8224\cdot RH-0.01888\cdot\frac{PH}{SH}+0.0009150\cdot D^2$$

$R^2=0.935$, $s_{y\cdot x}=0.4769$, n=1671

where $\hat{V}$ is predicted residual pulpwood volume and RH is the residual number of pulpwood sticks above and beyond sawtimber sticks.

Table 6 in the Appendix shows residual cubic-foot volumes for various numbers of sawtimber and residual pulpwood sticks based on Equation 4. Table 7 in the Appendix shows residual rough cord volumes for various numbers of sawtimber and residual pulpwood sticks. Volumes from Table 6 were divided by 79 cu.ft./cd. and then multiplied by 0.96 to obtain Table 7 values. A residual cubic-foot or rough cord volume table can also be developed from Equation 5 for various values of SH, RH, and D.

Method 2 -- Two prediction equations were developed for proportion of residual pulpwood volume in a sawtimber tree:

1. Height independent variables

(6) $$\hat{P}=-0.1943+0.9444\cdot\frac{RH}{PH}+0.1010\cdot\frac{1}{RH}+0.02757\cdot\frac{RH}{SH}$$

$R^2=0.990$, $s_{y\cdot x}=0.0215$, n=1671

2. Height and diameter independent variables

(7) $$\hat{P}=-0.2384+0.7804\cdot\frac{RH}{PH}+0.08480\cdot\frac{1}{RH}+0.03580\cdot\frac{RH}{SH}+1.2740\cdot\frac{1}{D}$$

$R^2=0.992$, $s_{y\cdot x}=0.0187$, n=1671

where $\hat{P}$ is the predicted proportion of residual pulpwood volume and SH is the number of sawtimber sticks. Table 8 in the Appendix gives values of $\hat{P}$ from Equation 6 for various numbers of sawtimber and residual pulpwood sticks.

Predicted residual pulpwood volume can be obtained by multiplying either Equation 6 or 7 times Equation 2. A residual pulpwood cubic-foot volume table can be developed for various values of SH, RH, and D. A separate table could be developed for each value of D.

For sawtimber trees with residual pulpwood volume, sawtimber cubic-foot volume can be obtained by subtracting predicted residual pulpwood volume (using Methods 1 or 2) from predicted pulpwood volume (Equation 2).

Validation

The data set used to validate the prediction equations consisted of the other approximately 20% of the trees in each of the 27 stands, yielding a total of 718 pulpwood trees. For each volume equation, the average relative error as a percent ($\overline{RE}$) was calculated for the pooled data set where

$$\overline{RE} = \sum_{i=1}^{n} RE_i \Big/ n$$

and $RE_i = [(\hat{V}_i - V_i)/V_i] \times 100$, $\hat{V}_i$ and V_i are the predicted and actual volumes for the i^{th} tree, and n is the number of trees in the pooled data set. The relative error as a percent for the sum of the predicted volumes was also calculated where

$$RE_S = \left[\left(\sum_{i=1}^{n} \hat{V}_i - \sum_{i=1}^{n} V_i \right) \Big/ \sum_{i=1}^{n} V_i \right] \times 100$$

and $\sum_{i=1}^{n} \hat{V}_i$ and $\sum_{i=1}^{n} V_i$ are the sum of the predicted and actual volumes, respectively.

For 718 trees, $\overline{RE}$ was 3.2% (range: -21.3 to 54.8%) and RE_S=0.45% for the total cubic-foot volume equation (Equation 1). RE_S's for the Gevorkiantz and Olsen (1955) and Buckman (1962) equations applied to the same trees were -6.80 and -9.35%, respectively.

For 716 trees, $\overline{RE}$ was 2.8% (range: -16.6 to 26.1%) and RE_S=0.72% for the pulpwood cubic-foot volume equation (Equation 2).

For 443 trees, $\overline{RE}$ was 0.36% (range: -13.1 to 65.7%) and RE_S=-0.29% for the sawtimber cubic-foot volume equations (Equation 3).

For 432 trees, $\overline{RE}$ was -0.07% (range: -40.9 to 33.6%) and 0.15% (range: -38.0 to 38.2%), and RE_S was -1.31% and -1.15%, for the residual pulpwood cubic-foot volume equations based on height (Equation 4) and height and diameter (Equation 5) independent variables, respectively (Method 1). $\overline{RE}$ was -1.26% (range: -72.8 to 127.6%) and -4.70% (range: -115.0 to 63.1%), and RE_S was -0.61% and -3.38%, for residual pulpwood cubic-foot volumes based on multiplying the proportion of

residual pulpwood volume Equation 6 (height independent variables) or Equation 7 (height and diameter independent variables), respectively, times the pulpwood volume Equation 2 (Method 2).

For residual pulpwood volume, the relative errors for the equations with height independent variables were, in general, somewhat smaller than the relative errors for the equations with height and diameter independent variables. The relative errors for the equations based on Method 1 were, in general, somewhat smaller than the relative errors for the equations based on Method 2.

One sample was taken in each of two red pine stands using 0.05-ac. circular plots. For each stand, DBH to the nearest 0.1 in. and merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter was measured for each tree with DBH $\geq$4.6 in. on each plot.

Four plots were selected from Stand 1 in the Lower Peninsula. DBH varied from 6.0 to 10.4 in., and merchantable height varied from 2-4 sticks. The average number of trees per plot was 24.8 (range: 20 to 30).

Five plots were selected from Stand 2 in the Upper Peninsula. DBH varied from 4.9 to 20.7 in., and merchantable height varied from 1-10 sticks. The average number of trees per plot was 11.6 (range: 8-14).

Volume in rough cords was estimated for both stands using the new pulpwood prediction Equation 2 (Table 5), Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Results for the two samples are shown in Table 9. For Stand 1, our new Equation yielded estimates 13.5 and 25.8% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively. For Stand 2, our new Equation yielded estimates 2.7% lower and 12.4% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively. Rough cord

Table 9. Volume estimates in rough cords per acre for the samples taken from 2 stands, one in the Lower Peninsula and one in the Upper Peninsula, based on our Equation 2 (Table 5), Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Stand	Equation 2	Gevorkiantz and Olsen (Table 6)	Ek and Droessler (Table 1)
1	42.0	37.0	33.4
2	87.7	90.2	78.1

volumes for 9-stick trees were obtained by adding 0.06 cords to rough cord volumes for 8-stick trees in Table 6. For one 19-in. tree, rough cord volume for 10 sticks was obtained by adding 0.05 cords to rough cord volume for 9 sticks.

As expected, the new pulpwood volume equations yielded estimates that were, in general, higher than those based on Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler. In comparing these results, remember that we used cubic-foot per rough cord values varying from 77 to 84 while Gevorkiantz and Olsen and Ek and Droessler used 79. Also, our approximate 3.6-in. minimum top diameter is different than the variable top diameters used by Gevorkiantz and Olsen and the 4-in. top diameter used by Ek and Droessler.

Guidelines for Users

The total cubic-foot volumes obtained using the new total volume Equation 1 (Table 2) will yield, in general, per acre estimates from 8-15% lower to 5-10% higher than estimates based on Table 3 of Gevorkiantz and Olsen, depending on stand DBH.

The rough cord volumes obtained using the new pulpwood volume equation (Table 5) will yield, in general, per acre estimates from 10 to 15% lower to 10

to 15% higher and from 10 to 25% higher than volume estimates from Gevorkiantz and Olsen (Table 6) and Ek and Droessler (Table 1), depending on stand DBH.

Validation results in terms of relative errors indicated that the residual pulpwood volume equations (Equations 4 and 6) based on height independent variables yielded slightly better results than the equations (Equations 5 and 7) based on height and diameter independent variables. For Method 1 (residual pulpwood volume equation), the relative errors were, in general, somewhat smaller than those for Method 2 (proportion of residual pulpwood volume equation times the pulpwood volume equation). Either equation of either method would yield adequate results for almost all situations. However, an argument can be made for Method 2. The use of Method 2 to obtain residual pulpwood volume followed by determining sawtimber volume as the difference between pulpwood volume and residual pulpwood volume yields a compatible approach to total merchantable volume estimation. The use of the sawtimber volume equation and the Method 1 residual pulpwood volume equations would not yield the total volume obtained by the pulpwood volume equation. However, our results show very little difference between these two approaches to estimate total volume from sawtimber and residual pulpwood volume estimates.

It should be noted that Method 1 based on the equation with height independent variables can be simplified considerably without sacrificing much accuracy. The residual cubic-foot (Table 6) and rough cord (Table 7) volumes are relatively constant over the number of sawtimber sticks for a given number of residual pulpwood sticks. Cubic-foot (rough cord) volume values of 2.4 (0.030), 4.2 (0.051), 6.0 (0.073), 7.8 (0.095), 9.6 (0.116), 11.3 (0.138), 13.1 (0.160), and 14.9 (0.181) would yield quick, accurate residual volume values for trees with 1, 2, 3, 4, 5, 6, 7, and 8 residual pulpwood sticks, respectively.

We recommend the use of the following cubic-foot volume equations for most cruising situations:

1. Total volume - Equation 1 (Table 2).
2. Pulpwood volume - Equation 2 (Table 3, rough cord volumes-Table 5).
3. Sawtimber volume - Equation 3 (Table 4).
4. Sawtimber and residual pulpwood volumes - Method 2 (height independent variables).
    A. Residual pulpwood cubic-foot volume - Equation 6 x Equation 2.
    B. Residual pulpwood rough cord volumes - Convert cubic-foot volumes to rough cord volumes.
    C. Sawtimber cubic-foot volume - subtract the product of Equation 6 and Equation 2 from Equation 2.

One might argue that Method 2 (height and diameter independent variables) based on Equations 7 and 2 be used instead of Equations 6 and 2 for those situations where somewhat more accuracy is needed for residual pulpwood volume. However, our results show that Equations 7 and 2 yield little, if any, increase in accuracy over Equations 6 and 2.

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Appendix

Table 2. Volume table showing total cu.ft. volume for various combinations of DBH and total height.

DBH (inches)	Total Height in Feet								
	20	30	40	50	60	70	80	90	100
5	1.4	2.2	3.0	3.9	4.8	5.7			
6	1.9	3.0	4.1	5.3	6.5	7.8			
7	2.5	3.9	5.4	6.9	8.5	10.1	11.8		
8		4.9	6.8	8.7	10.7	12.7	14.8	16.9	
9		6.0	8.3	10.6	13.1	15.6	18.1	20.7	23.3
10		7.2	9.9	12.7	15.7	18.6	21.7	24.7	27.9
11			11.7	15.0	18.4	21.9	25.5	29.1	32.8
12			13.5	17.4	21.4	25.5	29.6	33.8	38.1
13			15.5	20.0	24.6	29.2	34.0	38.8	43.7
14			17.6	22.7	27.9	33.2	38.6	44.1	49.6
15			19.9	25.5	31.4	37.3	43.4	49.6	55.8
16			22.2	28.5	35.0	41.7	48.5	55.4	62.4
17			24.6	31.7	38.9	46.3	53.8	61.5	69.2
18			27.1	34.9	42.9	51.0	59.3	67.8	76.3
19			29.8	38.3	47.1	56.0	65.1	74.4	83.8
20			32.5	41.8	51.4	61.1	71.1	81.2	91.5
21				45.5	55.9	66.5	77.3	88.3	99.4
22				49.2	60.5	72.0	83.7	95.6	107.7
23				53.1	65.3	77.7	90.3	103.2	116.2
24				57.2	70.2	83.6	97.2	110.9	125.0
25				61.3	75.3	89.6	104.2	119.0	134.1
26				65.6	80.6	95.9	111.5	127.3	143.4
27				70.0	85.9	102.3	118.9	135.8	153.0
28				74.5	91.5	108.9	126.6	144.6	162.8
29				79.1	97.1	115.6	134.4	153.5	172.9
30				83.8	103.0	122.5	142.5	162.7	183.3

Table 3. Volume table showing pulpwood cu.ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	1.1	2.0	2.9						
6	1.6	2.8	3.9	5.0					
7	2.0	3.7	5.1	6.5	7.8				
8	2.6	4.6	6.5	8.2	9.9	11.5			
9	3.2	5.6	7.9	10.1	12.1	14.1	16.1		
10	3.8	6.8	9.5	12.1	14.6	17.0	19.3		
11		8.0	11.2	14.3	17.2	20.0	22.8	25.5	
12		9.3	13.1	16.6	20.0	23.3	26.5	29.6	
13		10.7	15.0	19.1	23.0	26.7	30.4	34.0	
14		12.2	17.1	21.7	26.1	30.4	34.6	38.7	
15			19.2	24.4	29.4	34.3	39.0	43.6	48.1
16			21.5	27.3	32.9	38.3	43.6	48.8	53.8
17			23.9	30.4	36.6	42.6	48.5	54.2	59.8
18			26.4	33.5	40.4	47.0	53.5	59.8	66.0
19			29.0	36.8	44.4	51.7	58.8	65.7	72.5
20			31.7	40.3	48.5	56.5	64.2	71.8	79.2
21				43.8	52.8	61.5	69.9	78.2	86.2
22				47.5	57.2	66.6	75.8	84.7	93.5
23				51.3	61.8	72.0	81.9	91.5	101.0
24				55.2	66.5	77.5	88.1	98.5	108.7
25				59.3	71.4	83.2	94.6	105.8	116.7
26				63.5	76.5	89.0	101.3	113.2	124.9
27				67.8	81.6	95.1	108.1	120.9	133.4
28				72.2	87.0	101.2	115.2	128.7	142.0
29				76.7	92.4	107.6	122.4	136.8	151.0
30				81.4	98.0	114.1	129.8	145.1	160.1

Table 4. Volume table showing sawtimber cu.ft. volume for various combinations of DBH and merchantable height in 100-in. sticks to an approximate 7.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
9	3.6	6.6	9.4	12.1	14.7				
10	4.2	7.7	11.1	14.3	17.4	20.4			
11	4.9	9.0	12.9	16.6	20.2	23.7			
12	5.6	10.4	14.8	19.1	23.2	27.2	31.2		
13	6.4	11.8	16.8	21.6	26.3	30.9	35.4	39.8	
14	7.2	13.2	18.9	24.3	29.6	34.8	39.8	44.8	
15	8.0	14.7	21.1	27.2	33.0	38.8	44.4	50.0	55.5
16		16.3	23.4	30.1	36.6	43.0	49.2	55.4	61.4
17		18.0	25.7	33.1	40.3	47.3	54.2	61.0	67.6
18		19.7	28.2	36.3	44.2	51.8	59.4	66.8	74.1
19		21.5	30.7	39.5	48.1	56.5	64.7	72.8	80.7
20		23.3	33.3	42.9	52.2	61.3	70.2	78.9	87.6
21			36.0	46.3	56.4	66.2	75.8	85.3	94.6
22			38.7	49.9	60.7	71.3	81.7	91.8	101.9
23			41.6	53.5	65.2	76.5	87.6	98.6	109.3
24			44.5	57.3	69.7	81.9	93.8	105.4	117.0
25			47.4	61.1	74.4	87.3	100.0	112.5	124.8
26				65.0	79.2	93.0	106.5	119.7	132.8
27				69.1	84.1	98.7	113.0	127.1	141.0
28				73.2	89.1	104.6	119.8	134.7	149.4
29				77.4	94.2	110.6	126.6	142.4	158.0
30				81.6	99.4	116.7	133.6	150.3	166.7

Table 5. Volume table showing rough cord volume for various combinations of DBH and merchantable height in 100-in. sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	0.014	0.025	0.035						
6	0.020	0.034	0.048	0.060					
7	0.026	0.044	0.062	0.078	0.093				
8	0.032	0.056	0.078	0.099	0.117	0.135			
9	0.039	0.069	0.096	0.121	0.144	0.165	0.186		
10	0.047	0.082	0.116	0.145	0.173	0.199	0.223		
11		0.097	0.136	0.171	0.204	0.234	0.263	0.291	
12		0.113	0.159	0.199	0.237	0.273	0.306	0.338	
13		0.130	0.182	0.229	0.272	0.313	0.352	0.389	
14		0.148	0.207	0.260	0.310	0.356	0.400	0.442	
15			0.234	0.293	0.349	0.401	0.451	0.498	0.550
16			0.261	0.328	0.390	0.449	0.504	0.557	0.615
17			0.290	0.364	0.434	0.499	0.560	0.619	0.683
18			0.320	0.402	0.479	0.551	0.619	0.684	0.754
19			0.352	0.442	0.526	0.605	0.680	0.751	0.828
20			0.385	0.483	0.575	0.661	0.743	0.821	0.905
21				0.537	0.640	0.738	0.831	0.920	1.016
22				0.570	0.678	0.780	0.877	0.968	1.068
23				0.616	0.733	0.843	0.947	1.046	1.154
24				0.663	0.789	0.907	1.019	1.126	1.242
25				0.712	0.847	0.974	1.094	1.209	1.334
26				0.762	0.906	1.042	1.171	1.294	1.427
27				0.813	0.968	1.113	1.250	1.381	1.524
28				0.866	1.031	1.185	1.332	1.471	1.623
29				0.920	1.095	1.260	1.416	1.564	1.725
30				0.976	1.162	1.336	1.501	1.658	1.830

Table 6. Residual pulpwood cu.ft. volume for various numbers of sawtimber and residual pulpwood sticks based on Equation 4.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	2.40	4.16	5.92	7.68	9.44	11.20	12.95	14.71
2	2.42	4.19	5.97	7.74	9.52	11.29	13.07	14.84
3	2.43	4.21	5.99	7.77	9.55	11.33	13.11	14.89
4	2.43	4.21	5.99	7.78	9.56	11.34	13.13	14.91
5	2.43	4.21	6.00	7.78	9.57	11.35	13.14	14.92
6	2.43	4.22	6.00	7.79	9.57	11.36	13.15	14.93
7	2.43	4.22	6.00	7.79	9.58	11.36	13.15	14.94
8	2.43	4.22	6.01	7.79	9.58	11.37	13.16	14.94

Table 7. Residual rough cord volume for various numbers of sawtimber and residual pulpwood sticks based on Equation 4.

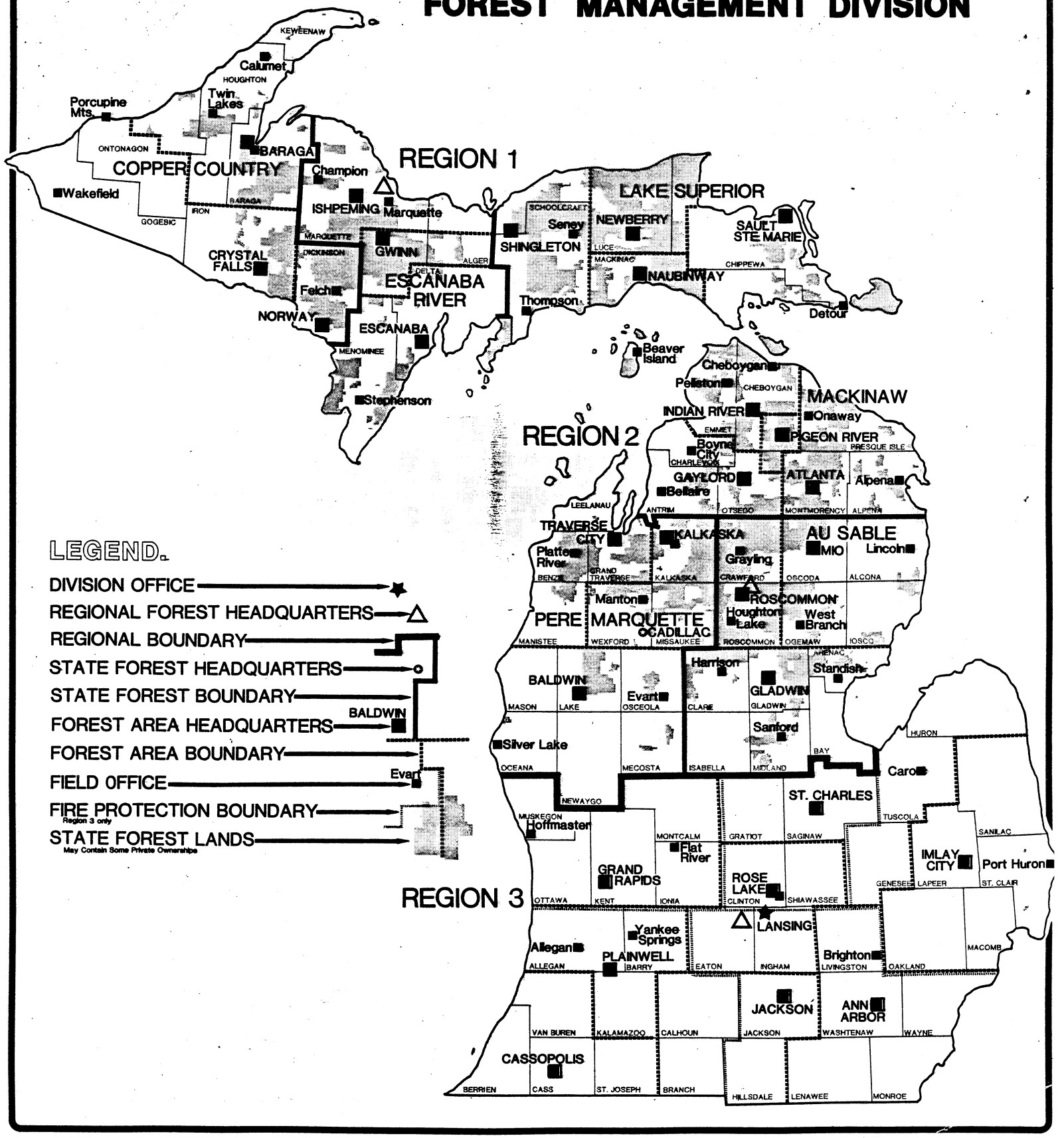
Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.029	0.051	0.072	0.093	0.115	0.136	0.157	0.179
2	0.029	0.051	0.073	0.094	0.116	0.137	0.159	0.180
3	0.029	0.051	0.073	0.094	0.116	0.138	0.159	0.181
4	0.030	0.051	0.073	0.095	0.116	0.138	0.160	0.181
5	0.030	0.051	0.073	0.095	0.116	0.138	0.160	0.181
6	0.030	0.051	0.073	0.095	0.116	0.138	0.160	0.181
7	0.030	0.051	0.073	0.095	0.116	0.138	0.160	0.182
8	0.030	0.051	0.073	0.095	0.116	0.138	0.160	0.182

Table 8. Proportions of residual pulpwood cu.ft. volume in a sawtimber tree for various numbers of sawtimber and residual pulpwood sticks based on Equation 6.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.41	0.54	0.63	0.70	0.75	0.80	0.84	0.88
2	0.24	0.36	0.45	0.52	0.57	0.61	0.65	0.68
3	0.15	0.25	0.34	0.41	0.46	0.51	0.55	0.58
4	0.10	0.18	0.26	0.33	0.38	0.43	0.47	0.50
5	0.07	0.14	0.21	0.27	0.33	0.37	0.41	0.44
6	0.05	0.10	0.17	0.23	0.28	0.32	0.36	0.39
7	0.03	0.07	0.13	0.19	0.24	0.28	0.32	0.35
8	0.01	0.05	0.11	0.16	0.21	0.25	0.28	0.32

MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES
FOREST MANAGEMENT DIVISION



LEGEND.

- DIVISION OFFICE
- REGIONAL FOREST HEADQUARTERS
- REGIONAL BOUNDARY
- STATE FOREST HEADQUARTERS
- STATE FOREST BOUNDARY
- FOREST AREA HEADQUARTERS — BALDWIN
- FOREST AREA BOUNDARY
- FIELD OFFICE — Evart
- FIRE PROTECTION BOUNDARY — Region 3 only
- STATE FOREST LANDS — May Contain Some Private Ownership