# BARK FACTOR EQUATIONS 

## FOR OAK IN MICHIGAN



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by

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## BACKGROUND

Bark factor (BF) is the ratio of diameter inside bark (DIB) to diameter outside bark (DOB) at a given tree height. Even though bark factor does increase with height for many species, a constant bark factor, usually determined at breast height, has been assumed, in many cases, for all tree heights for many species. Thus, the use of a constant bark factor for all tree heights will usually lead to underestimates of most tree and log solid wood volumes and overestimates of bark volume for many species.

Bark factor equations have been developed for aspen (Fowler and Hussain 1987b, Fowler 1991), jack pine (Fowler and Hussain 1991, Fowler 1993), and red pine (Fowler and Hussain 1987a, Fowler and Damschroder 1988) in Michigan where bark factor was regressed on tree height (TH). In all cases, there was a very strong relationship between BF and TH. Bark factor equations were also developed for paper birch (Fowler and Hussain 1997) in Michigan where BF was regressed on TH and DOB. Both relationships were relatively weak with the relationship to DOB being somewhat stronger.

## PURPOSE

The purpose of this paper is to present bark factor prediction equations for black oak, red oak, and white oak in Michigan and show how the prediction equations may be used.

## METHODS AND MATERIALS

As part of a larger study to develop new volume equations for hardwoods in Michigan, felled tree measurements were made on a total of 53 red oak trees ( 51 trees and two trees from two stands, respectively, from the Mackinaw State Forest) and 23 black oak and 28 white oak trees from a stand in the Pere Marquette State Forest during May-August, 1995. DIB and DOB were measured to the nearest 0.01 in . at stump height, which varied from $1-41 \mathrm{in}$., at the top of each $8.3-\mathrm{ft}$. bolt ( $100-\mathrm{in}$. stick), or other nominal bolt length varying from $6-16 \mathrm{ft}$.), cut out of the stem of each tree to an approximate $3.6-\mathrm{in}$. diameter top limit (i.e., stemwood), and at the bottom and top of each 8.3 ft . bolt cut out of any limbs and top forks of each tree to an approximate 3.6-in. diameter top limit (i.e., topwood). DBH was measured to the nearest 0.1 in ., and bark thickness at DBH height was measured to the nearest 0.01 in . DBH height was 4.5 ft . from the ground except for trees forked below 4.5 ft . where DBH height was approximately 4.5 ft . above the fork. DBH varied from 3.7-24.6 in. for the data set of 104 trees.

## Stemwood

The prediction data set included (1) 23 black oak trees from the Pere Marquette stand, (2) 53 red oak trees ( 51 and two trees from the two Mackinaw stands, respectively), and (3) 28 white oak trees from the Pere Marquette stand. This yielded 146, 340, and 171 bark factor measurements for black, red, and white oak, respectively.

The mean, minimum, and maximum DBH in in. and merchantable height (MH) in ft. for the trees of each species are shown below. MH is the height of the tree from the ground to an approximate $3.6-\mathrm{in}$. diameter top limit.

|  |  | DBH |  |  | MH |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Species | No. of Trees | $\bar{x}$ | Min.- Max. |  | $\bar{x}$ | Min. -Max. |  |
| BO | 23 | 15.1 | $10.7-20.0$ |  | 39.08 | $9.00-61.92$ |  |
| RO | 53 | 10.5 | $3.7-24.2$ |  | 37.37 | $8.67-59.58$ |  |
| WO | 28 | 13.9 | $10.1-24.6$ |  | 37.07 | $10.92-56.67$ |  |

The following table shows the mean, minimum, and maximum BF, tree height to measurement in ft . (TH), and DOB at TH for the set of bark factor measurements for each species.

| Species | No. of BF Measurements | BF |  | TH |  | DOB at TH |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\bar{x}$ | Min.- Max. | $\bar{x}$ | Min. -Max. | $\bar{\chi}$ | Min. -Max. |
| BO | 146 | 0.964 | 0.920-0.996 | 20.12 | 0.33-61.92 | 12.43 | 3.61-26.34 |
| RO | 340 | 0.976 | 0.916-0.998 | 17.94 | 0.25-59.58 | 8.69 | 1.90-26.70 |
| WO | 171 | 0.955 | 0.917-0.992 | 18.93 | 0.08-56.67 | 11.30 | 3.22-29.88 |

## Topwood

The prediction data set included 19, 24, and 20 of the total of 23 black, 53 red, and 28 white oak trees, respectively. This yielded 162, 207, and 214 bark factor measurements for black, red, and white oak, respectively.

The mean, minimum, and maximum DBH in in., MH in ft., and number of topwood sticks for the trees of each species are shown below.

| Species | No. of Trees | DBH |  | MH |  | No. of topwood sticks |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\bar{x}$ | Min.-Max. | $\bar{x}$ | Min. -Max. | $\bar{x}$ | Min.-Max. |
| BO | 19 | 15.9 | 12.5-20.0 | 36.18 | 9.00-61.92 | 5.4 | 1-10 |
| RO | 24 | 14.1 | 8.6-24.2 | 40.72 | 10.50-59.58 | 5.2 | 1-40 |
| WO | 20 | 14.9 | 10.8-24.6 | 32.21 | 10.92-56.67 | 6.6 | 1-35 |

The following table shows the mean, minimum, and maximum BF and DOB at the BF measurement point for the set of bark factor measurements for each species.

| Species | No. of BF <br> Measurements | BF |  |  | DOB |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\bar{x}$ | Min.-Max. |  | $\bar{x}$ | Min. -Max. |
|  | 162 | 0.974 | $0.931-0.996$ |  | 6.78 | $3.26-17.55$ |
| RO | 207 | 0.986 | $0.949-0.997$ |  | 4.93 | $1.99-12.35$ |
| WO | 214 | 0.962 | $0.914-0.996$ |  | 5.88 | $3.18-15.51$ |

## RESULTS

The best prediction equations, based on simplicity, meeting the assumptions of normality and homogeneity, and having among the smallest standard errors of the estimate (sy.x) and the largest coefficients of determination ( $\mathrm{R}^{2}$ ), were:

## Stemwood

Black Oak ( $n=146$ )

|  |  | $\mathrm{R}^{2}$ |  |
| :--- | :--- | :--- | :--- |
| (1) $\hat{\mathrm{BF}}=0.981363-0.001396 \bullet \mathrm{DOB}$ | 0.270 | 0.011927 |  |
| (2) $\hat{\mathrm{BF}}=0.954401+0.004136 \bullet \ln \mathrm{TH}$ | 0.190 | 0.012559 |  |
| (3) $\hat{\mathrm{BF}}=0.979283-0.001303 \bullet \mathrm{DOB}+0.000401 \bullet \ln \mathrm{TH}$ | 0.270 | 0.011964 |  |

Red Oak ( $n=340$ )
(4) $\hat{\mathrm{BF}}=0.997708-0.002445 \cdot \mathrm{DOB}$

| $\frac{\mathrm{R}^{2}}{0.418}$ |  |
| :---: | :---: |
| 0.275 |  |
| 0.012746 |  |
| 0.014227 |  |

(5) $\hat{\mathrm{BF}}=0.962791+0.006068 \bullet \ln \mathrm{TH}$
$0.275 \quad 0.014227$
(6) $\begin{array}{r}\hat{\mathrm{BF}}=0.986153-0.001939\end{array}$ DOB $+0.003177 \bullet 1$ White Oak $(n=171)$
$\begin{array}{llll}\text { (7) } \hat{\mathrm{BF}}=0.958801-0.000349 \bullet \mathrm{DOB} & \mathrm{R}^{2} & & \mathrm{~s}_{\mathrm{y} \bullet \mathrm{x}} \\ (8) \hat{\mathrm{BF}}=0.952426+0.001065 \bullet \ln \mathrm{TH} & 0.014 & 0.015007 \\ (9) \hat{\mathrm{BF}}=0.957163-0.000276 \bullet \mathrm{DOB}+0.000357 \bullet \ln \mathrm{TH} & 0.015 & 0.015048\end{array}$

Prediction Equations 1, 4, and 7 for BO, RO, and WO, respectively, yield the following estimated bark factors.

Prediction Equations 1, 4, and 7

| DOB |  | $\hat{B F}$ |  | DOB | $\hat{B F}$ |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO | (in.) | BO | RO | WO |
| 3.0 | 0.977 | 0.990 | 0.958 | 17.0 | 0.958 | 0.956 | 0.953 |
| 4.0 | 0.976 | 0.988 | 0.957 | 18.0 | 0.956 | 0.954 | 0.953 |
| 5.0 | 0.974 | 0.985 | 0.957 | 19.0 | 0.955 | 0.951 | 0.952 |
| 6.0 | 0.973 | 0.983 | 0.957 | 20.0 | 0.953 | 0.949 | 0.952 |
| 7.0 | 0.972 | 0.981 | 0.956 | 21.0 | 0.952 | 0.946 | 0.951 |
| 8.0 | 0.970 | 0.978 | 0.956 | 22.0 | 0.951 | 0.943 | 0.951 |
| 9.0 | 0.969 | 0.976 | 0.956 | 23.0 | 0.949 | 0.941 | 0.951 |
| 10.0 | 0.967 | 0.973 | 0.955 | 24.0 | 0.948 | 0.939 | 0.950 |
| 11.0 | 0.966 | 0.971 | 0.955 | 25.0 | 0.946 | 0.937 | 0.950 |
| 12.0 | 0.965 | 0.968 | 0.955 | 26.0 | 0.945 | 0.934 | 0.950 |
| 13.0 | 0.963 | 0.966 | 0.954 | 27.0 | 0.944 | 0.932 | 0.949 |
| 14.0 | 0.962 | 0.963 | 0.954 | 28.0 | 0.942 | 0.929 | 0.949 |
| 15.0 | 0.960 | 0.961 | 0.954 | 29.0 | 0.941 | 0.927 | 0.949 |
| 16.0 | 0.959 | 0.959 | 0.953 | 30.0 | 0.939 | 0.924 | 0.948 |

Predicted Equations 2, 5, and 8 for BO, RO, and WO, respectively, yield the following estimated bark factors.

Prediction Equations 2, 5, and 8

| TH <br> (ft.) | $\hat{B F}$ |  |  | TH <br> (ft.) | $\hat{B F}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO |  | BO | RO | WO |
| 0.25 | 0.949 | 0.954 | 0.951 | 25.5 | 0.968 | 0.982 | 0.956 |
| 0.5 | 0.952 | 0.959 | 0.952 | 34.0 | 0.969 | 0.984 | 0.956 |
| 1.0 | 0.954 | 0.963 | 0.952 | 42.5 | 0.970 | 0.986 | 0.956 |
| 2.0 | 0.957 | 0.967 | 0.953 | 51.0 | 0.971 | 0.987 | 0.957 |
| 3.0 | 0.959 | 0.969 | 0.954 | 59.5 | 0.971 | 0.988 | 0.957 |
| 4.5 | 0.961 | 0.972 | 0.954 | 68.0 | 0.972 | 0.988 | 0.957 |
| 8.5 | 0.963 | 0.976 | 0.955 | 76.5 | 0.972 | 0.989 | 0.957 |
| 17.0 | 0.966 | 0.980 | 0.955 |  |  |  |  |

The predicted BF based on Equations 1, 4, and 7 varies from 0.977 for $\mathrm{DOB}=3.0$ in. to 0.939 for $\mathrm{DOB}=30.0$ in. (range $=0.038$ ), 0.990 for $\mathrm{DOB}=3.0$ in. to 0.924 for $\mathrm{DOB}=30.0$ in. (range $=0.066$ ), and 0.958 for $\mathrm{DOB}=3.0$ in. to 0.948 for $\mathrm{DOB}=30.0 \mathrm{in}$. (range $=0.010$ ), respectively. The ranges of predicted BF values based on Equations 2, 5, and 8 are considerably smaller, being $0.23,0.35$,
and 0.08 for black, red, and white oak, respectively. Because of these moderate ranges and the low $R^{2}$ values of the prediction equations, you might argue that the mean bark factor yields an adequate prediction model.
$\begin{array}{lll}\text { (10) } & \mathrm{BO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{146} B F_{i} / 146=0.964 \\ \text { (11) } & \mathrm{RO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{340} B F_{i} / 340=0.976 \\ \text { (12) } \mathrm{WO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{171} B F_{i} / 171=0.955 & 0.016686 \\ \end{array}$

See the above two tables to find where Equations 10, 11, and 12 over- and underestimate related to Equations 1-2, 4-5, and 7-8, respectively.

Prediction Equations 1, 4, and 7 are significantly different (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}<0.01$; F-test for equal slopes, $\mathrm{p}<0.001$ ). Prediction Equations 2, 5, and 8 are significantly different (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}<0.10$; F-test for equal slopes, $\mathrm{p}<0.001$ ). Prediction Equations 10,11 , and 12 related to mean bark factors are also significantly different ( F -test for equal variances, $\mathrm{p}=0.029$; F -test for equal means, $\mathrm{p}<0.001$ ). All Bonferroni pairwise comparisons of means are significantly different ( $\mathrm{p}<0.001$ ).

## Topwood

(13) $\mathrm{BO}: \hat{\mathrm{BF}}=1.000578-0.003896 \bullet \mathrm{DOB}$

| $\mathrm{R}^{2}$ | $\mathrm{~s}_{\mathrm{y} \bullet \mathrm{x}}$ |
| :---: | :---: |
| 0.452 |  |
| 0.011531 |  |
| 0.273 |  |
| 0.219 | 0.007826 |
|  | 0.014579 |

Prediction Equations 13, 14, and 15 for BO, RO, and WO, respectively, yield the following estimated bark factors.

Prediction Equations 13, 14, and 15

| $\begin{gathered} \text { DOB } \\ \text { (in.) } \end{gathered}$ | $\hat{B F}$ |  |  | $\begin{gathered} \text { DOB } \\ \text { (in.) } \\ \hline \end{gathered}$ | $\hat{B F}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO |  | BO | RO | WO |
| 3.0 | 0.989 | 0.991 | 0.971 | 12.0 | 0.954 | 0.969 | 0.942 |
| 4.0 | 0.985 | 0.989 | 0.968 | 13.0 | 0.950 | 0.967 | 0.939 |
| 5.0 | 0.981 | 0.986 | 0.965 | 14.0 | 0.946 | 0.964 | 0.936 |
| 6.0 | 0.977 | 0.984 | 0.962 | 15.0 | 0.942 | 0.962 | 0.933 |
| 7.0 | 0.973 | 0.981 | 0.959 | 16.0 | 0.938 | 0.959 | 0.929 |
| 8.0 | 0.969 | 0.979 | 0.955 | 17.0 | 0.934 | 0.957 | 0.926 |
| 9.0 | 0.966 | 0.976 | 0.952 | 18.0 | 0.930 | 0.954 | 0.923 |
| 10.0 | 0.962 | 0.974 | 0.949 | 19.0 | 0.927 | 0.952 | 0.920 |
| 11.0 | 0.958 | 0.971 | 0.945 | 20.0 | 0.923 | 0.949 | 0.916 |

The predicted BF based on Equations 13, 14, and 15 varies from 0.989 for $\mathrm{DOB}=3.0$ in. to 0.923 for $\mathrm{DOB}=20.0$ in. (range $=0.066$ ), 0.991 at $\mathrm{DOB}=3.0$ in. to 0.949 at $\mathrm{DOB}=20.0$ in. (range $=0.042$ ), and 0.971 at $\mathrm{DOB}=3.0 \mathrm{in}$. to 0.916 for $\mathrm{DOB}=20.0 \mathrm{in}$. (range $=0.055$ ). Because of these moderate ranges and the low $R^{2}$ values of the prediction equations, you might argue that the mean bark factor yields an adequate prediction model.

$$
\begin{array}{llc} 
& & \mathrm{s}_{\mathrm{y} \bullet \mathrm{x}} \\
\mathrm{BO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{162} B F_{i} / 162=0.974 & 0.015525 \\
\text { RO: } & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{207} B F_{i} / 207=0.986 & 0.009157  \tag{18}\\
\text { WO: } & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{214} B F_{i} / 214=0.962 & 0.016463
\end{array}
$$

See the above table to find where Equations 16, 17, and 18 over- and underestimate related to Equations 13, 14, and 15 , respectively.

Prediction Equations 13, 14, and 15 are significantly different (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}<0.001$; F-test for equal slopes, $\mathrm{p}=0.028$ ). Prediction Equations 16, 17, and 18 related to mean bark factors are also significantly different (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}<0.001$; F-test for equal means, $\mathrm{p}<0.001$ ). All Bonferroni pairwise comparisons of means are significantly different ( $\mathrm{p}<0.001$ ).

## Pooled prediction equations

The stemwood and topwood BF prediction equations with DOB as the independent variable are significantly different for black oak (F-test for equal variances, $\mathrm{p}>0.25$; F-test for equal slopes, $\mathrm{p}<0.001$ ), not significantly differently for red oak (F-test for equal variances, $\mathrm{p}>0.25$; F-test for equal slopes, $\mathrm{p}=0.995$; F-test for equal intercepts, $\mathrm{p}=0.520$ ), and significantly different for white oak (F-test for equal variances, $\mathrm{p}>0.25$; F -test for equal slopes $<0.001$ ). The two equations for red oak can be pooled. If the two equations for black or white oak are pooled, some prediction accuracy will be lost.

The pooled prediction equations are:

| $n$ | $n$ |  | $\mathrm{R}^{2}$ |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $\mathrm{~s}_{\mathrm{y} \bullet \mathrm{x}}$ |  |  |
| $\mathrm{BO}:$ | $\hat{\mathrm{BF}}=0.987441-0.001912 \bullet \mathrm{DOB}$ | 308 | 0.366 |  |
| 0.012442 |  |  |  |  |
| $\mathrm{RO}:$ | $\hat{\mathrm{BF}}=0.998246-0.002482 \bullet \mathrm{DOB}$ | 547 | 0.457 | 0.011131 |
| WO: $\hat{\mathrm{BF}}=0.968069-0.001095 \cdot \mathrm{DOB}$ | 385 | 0.100 | 0.015449 |  |

Prediction Equations 19, 20, and 21 for BO, RO, and WO, respectively, yield the following estimated bark factors.

Prediction Equations 19, 20, and 21

| $\begin{gathered} \text { DOB } \\ \text { (in.) } \end{gathered}$ | $\hat{B F}$ |  |  | $\begin{gathered} \text { DOB } \\ \text { (in.) } \end{gathered}$ | $\hat{B F}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO |  | BO | RO | Wo |
| 3.0 | 0.982 | 0.991 | 0.965 | 17.0 | 0.955 | 0.956 | 0.949 |
| 4.0 | 0.980 | 0.988 | 0.964 | 18.0 | 0.953 | 0.954 | 0.948 |
| 5.0 | 0.978 | 0.986 | 0.963 | 19.0 | 0.951 | 0.951 | 0.947 |
| 6.0 | 0.976 | 0.983 | 0.961 | 20.0 | 0.949 | 0.949 | 0.946 |
| 7.0 | 0.974 | 0.981 | 0.960 | 21.0 | 0.947 | 0.946 | 0.945 |
| 8.0 | 0.972 | 0.978 | 0.959 | 22.0 | 0.945 | 0.944 | 0.944 |
| 9.0 | 0.970 | 0.976 | 0.958 | 23.0 | 0.943 | 0.941 | 0.943 |
| 10.0 | 0.968 | 0.973 | 0.957 | 24.0 | 0.942 | 0.939 | 0.942 |
| 11.0 | 0.966 | 0.971 | 0.956 | 25.0 | 0.940 | 0.936 | 0.941 |
| 12.0 | 0.964 | 0.968 | 0.955 | 26.0 | 0.938 | 0.934 | 0.940 |
| 13.0 | 0.963 | 0.966 | 0.954 | 27.0 | 0.936 | 0.931 | 0.939 |
| 14.0 | 0.961 | 0.963 | 0.953 | 28.0 | 0.934 | 0.929 | 0.937 |
| 15.0 | 0.959 | 0.961 | 0.952 | 29.0 | 0.932 | 0.926 | 0.936 |
| 16.0 | 0.957 | 0.959 | 0.951 | 30.0 | 0.930 | 0.924 | 0.935 |

Note that the BF estimates for RO are very close to those of Equation 4 for stemwood and Equation 14 for topwood. BF estimates for BO are (1) higher than those of Equation 1 for $\mathrm{DOB}<11.0 \mathrm{in}$. and lower for $\mathrm{DOB}>11.0 \mathrm{in}$., and (2) lower than those of Equation 13 for $\mathrm{DOB} \leq 6.0 \mathrm{in}$. and higher for $\mathrm{DOB} \geq 7.0 \mathrm{in}$. BF estimates for WO are (1) higher than those of Equation 7 for $\mathrm{DOB}<13.0 \mathrm{in}$. and lower for $\mathrm{DOB}>13.0 \mathrm{in}$., and (2) lower than those of Equation 15 for $\mathrm{DOB} \leq 6.0 \mathrm{in}$. and higher for $\mathrm{DOB} \geq 7.0 \mathrm{in}$.

The predicted BF based on Equations 19, 20, and 21 varies from 0.982 for $\mathrm{DOB}=3.0 \mathrm{in}$. to 0.930 for $\mathrm{DOB}=30.0 \mathrm{in}$. (range $=0.052$ ), 0.991 for $\mathrm{DOB}=3.0 \mathrm{in}$. to 0.924 for $\mathrm{DOB}=30.0 \mathrm{in}$. (range $=0.067$ ), and 0.965 for $\mathrm{DOB}=3.0$ in. to 0.935 (range $=0.030$ ), respectively. Because of these moderate ranges and the low $\mathrm{R}^{2}$ values of the prediction equations, you might argue that the mean bark factor yields an adequate prediction model.

The stemwood and topwood mean BFs are significantly different for black oak (F-test for equal variances, $\mathrm{p}=0.177$; F -test for equal means, $\mathrm{p}<0.001$ ), red oak ( F -test for equal variances, $\mathrm{p}<0.001$; F-test for equal means, $\mathrm{p}<0.001$ ), and white oak ( F -test for equal variances, $\mathrm{p}=0.227$;

F-test for equal means, $\mathrm{p}<0.001$ ). If the stemwood and topwood bark factors are pooled, some prediction accuracy will be lost.

The pooled mean bark factors are:
$\begin{array}{lll}\text { (22) } & \mathrm{BO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{308} B F_{i} / 308=0.969 \\ & 0.015606 \\ \text { (23) } & \mathrm{RO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{547} B F_{i} / 547=0.980 \\ \text { (24) } & \mathrm{WO}: & \hat{\mathrm{BF}}=\overline{\mathrm{BF}}=\sum_{i=1}^{385} B F_{i} / 385=0.015084\end{array}$

See the above table to find where Equations 22, 23, and 24 over- and underestimate related to Equations 19, 20, and 21, respectively.

## Bark thickness

For the stemwood data set ( $n=657$ ), average bark thickness (BT) was significantly different for the three oak species (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}=0.396$; F-test for equal means, $\mathrm{p}=0.001$ ). Average bark thickness was 0.485 in. (min. $=0.015$, max. $=1.285$ ) for black oak, 0.252 in. $(\min .=0.010, \max .=1.200)$ for red oak, and $0.519(\min .=0.035, \max .=1.600)$ for white oak. BT was positively related to $\mathrm{DOB}(\mathrm{BO}: \mathrm{r}=0.873 ; \mathrm{RO}: \mathrm{r}=0.858$; WO: $\mathrm{r}=0.815$ ) with $\mathrm{p}<0.001$ for each species. BT was positively related to $\mathrm{DBH}(\mathrm{BO}: \mathrm{r}=0.429 ; \mathrm{RO}: \mathrm{r}=0.502 ; \mathrm{WO}: \mathrm{r}=0.534$ ) and negatively related to $\mathrm{TH}(\mathrm{BO}: \mathrm{r}=-0.814 ; \mathrm{RO}: \mathrm{r}=-0.527$; WO: $\mathrm{r}=-0.686)$ with $\mathrm{p}<0.001$ in each case. Average BTs for various DOB and TH classes for the three oak species are as follows.

| DOB Class (in.) | BT |  |  | TH Class <br> (ft.) | BT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO |  | BO | RO | WO |
| $\leq 5.00$ | 0.054 | 0.037 | 0.121 | $\leq 0.50$ | 0.731 | 0.410 | 0.699 |
| 5.01 to 10.00 | 0.211 | 0.152 | 0.379 | 0.51 to 4.50 | 0.711 | 0.439 | 0.722 |
| 10.01 to 15.00 | 0.535 | 0.470 | 0.589 | 4.51 to 10.00 | 0.650 | 0.260 | 0.561 |
| 15.01 to 20.00 | 0.730 | 0.733 | 0.737 | 10.01 to 20.00 | 0.547 | 0.271 | 0.570 |
| $>20.00$ | 0.900 | 0.828 | 1.080 | 20.01 to 30.00 | 0.453 | 0.155 | 0.472 |
|  |  |  |  | 30.01 to 40.00 | 0.296 | 0.084 | 0.326 |
|  |  |  |  | 40.01 to 50.00 | 0.102 | 0.040 | 0.183 |
|  |  |  |  | >50.00 | 0.083 | 0.044 | 0.101 |

BT is smallest for RO with WO having somewhat larger BT than BO. In general, BT increases with DOB and decreases with TH .

For the topwood data set ( $n=583$ ), BT was significantly different for the three oak species (Bartlett's $\chi^{2}$-test for equal variances, $\mathrm{p}<0.001$; F-test for equal means, $\mathrm{p}=0.001$ ). Average BT was 0.203 in . ( $\min =0.015, \max .=0.840)$ for black oak, 0.077 in . $(\min .=0.010, \max .=0.555)$ for red oak, and 0.240 in . $(\min .=0.015, \max =0.900)$ for white oak. BT was positively related to DOB (BO: $\mathrm{r}=0.888$; RO: $\mathrm{r}=0.771$; WO: $\mathrm{r}=0.877$ with $\mathrm{p}<0.001$ for each species) and DBH (BO: $\mathrm{r}=0.303, \mathrm{p}<0.001$; RO: $\mathrm{r}=0.149, \mathrm{p}=0.032$; WO: $\mathrm{r}=0.235, \mathrm{p}<0.001$ ). Average BTs for various DOB classes for the three oak species are as follows:

| DOB Class <br> (in.) | BT |  |  |
| ---: | :---: | :---: | :---: |
|  | BO | RO | WO |
| $\leq 5.00$ | 0.051 | 0.040 | 0.122 |
| 5.01 to 7.00 | 0.140 | 0.077 | 0.243 |
| 7.01 to 9.00 | 0.268 | 0.211 | 0.389 |
| 9.01 to 11.00 | 0.428 | 0.294 | 0.530 |
| 11.01 to 13.00 | 0.559 | 0.332 | 0.632 |
| 13.01 to 15.00 | 0.583 | - | 0.762 |
| 15.01 to 17.00 | 0.635 | - | 0.780 |
| $>17.00$ | 0.760 | - | - |

BT is smallest for RO with WO having larger BT than BO . BT increases with DOB.

## Comparison with other BF equations

Fowler (1993) showed that while there were significant species differences between BF equations for aspen, jack pine, and red pine, there was a very strong relationship between BF and tree height for each species (i.e., $\mathrm{R}^{2}>0.97$ for each species). BF was a function of TH and $\ln \mathrm{TH}$, showing that BF increased with TH to some maximum and then decreased for larger THs with the steepness of the decrease depending on the species. For all three species, BF was not strongly related to DBH or DOB at a given TH.

For paper birch (Fowler and Hussain 1997), BF significantly increased with DOB at TH $\left(\mathrm{R}^{2}=0.219\right)$ and significantly decreased with $\ln T H\left(\mathrm{R}^{2}=0.166\right)$ for stemwood, while $B F$ significantly increased with DOB for topwood $\left(\mathrm{R}^{2}=0.218\right)$. BF was much more variable than for aspen, red pine, and jack pine.

This study shows that the three oak species have BFs that are quite variable and prediction equations with the same independent variables as for paper birch. For stemwood, BF decreased with DOB and increased with TH, while for topwood BF decreased with DOB. These prediction equations were significant, but they were only moderately strong at best, being only somewhat stronger, in general, than the prediction equations for paper birch. The BF equations were significantly different for the three species, and for topwood versus stemwood except for red oak.

## GUIDELINES FOR USERS

We recommend use of the following equations for black, red, and white oak when accurate estimates of bark factors are desired:

## Stemwood

- Black oak
(1) $\hat{\mathrm{BF}}=0.981363-0.001396 \cdot \mathrm{DOB}$
(2) $\hat{\mathrm{BF}}=0.954401+0.004136 \cdot \ln \mathrm{TH}$
- Red oak
(3) $\hat{\mathrm{BF}}=0.997708-0.002445 \cdot \mathrm{DOB}$
(4) $\hat{\mathrm{BF}}=0.962791+0.006068 \bullet \ln \mathrm{TH}$
- White oak
(5) $\hat{\mathrm{BF}}=0.958801-0.000349 \bullet \mathrm{DOB}$
(6) $\hat{\mathrm{BF}}=0.952426+0.001065 \bullet \ln \mathrm{TH}$

Use Equations 1, 3, and 5 if DOB is measured. Use Equations 2, 4, and 6 when only TH is measured.

## Topwood

- (7) $\mathrm{BO}: \hat{\mathrm{BF}}=1.000578-0.003896 \cdot \mathrm{DOB}$
- (8) RO: $\hat{\mathrm{BF}}=0.998427-0.002448 \cdot \mathrm{DOB}$
- (9) WO: $\hat{\mathrm{BF}}=0.981399-0.003250 \bullet \mathrm{DOB}$

The equation for stemwood and topwood pooled could be used if DOB is measured with moderate loss in accuracy for black and white oak and little loss in accuracy for red oak. The pooled equations for black and white oak are more accurate for stemwood compared to topwood.

- (10) $\mathrm{BO}: \hat{\mathrm{BF}}=0.987441-0.001912 \bullet \mathrm{DOB}$
- (11) RO: $\hat{\mathrm{BF}}=0.998246-0.002482 \bullet$ DOB
- (12) WO: $\hat{\mathrm{BF}}=0.968069-0.001095 \cdot \mathrm{DOB}$

For reasonable accuracy in many situations, the following constants could be used for bark factors.

| DOB Class <br> (in.) | Stemwood |  |  | Topwood |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BO | RO | WO | BO | RO | WO |
| DOB $\leq 5.0$ | 0.976 | 0.988 | 0.957 | 0.985 | 0.989 | 0.968 |
| $5.0<\mathrm{DOB} \leq 10.0$ | 0.970 | 0.978 | 0.956 | 0.969 | 0.979 | 0.955 |
| $10.0<\mathrm{DOB} \leq 15.0$ | 0.963 | 0.966 | 0.955 | 0.950 | 0.967 | 0.939 |
| $15.0<\mathrm{DOB} \leq 20.0$ | 0.956 | 0.954 | 0.953 | 0.930 | 0.954 | 0.923 |
| DOB>20.0 | 0.946 | 0.936 | 0.950 | 0.910 | 0.937 | 0.907 |

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| TH <br> (ft.) | Stemwood |  |  |
| ---: | :---: | :---: | :---: |
|  | BO | RO | WO |
| $\mathrm{TH} \leq 0.5$ | 0.951 | 0.956 | 0.952 |
| $0.5<\mathrm{TH} \leq 4.5$ | 0.958 | 0.968 | 0.953 |
| $4.5<\mathrm{TH} \leq 10.0$ | 0.962 | 0.974 | 0.954 |
| $10.0<\mathrm{TH} \leq 20.0$ | 0.966 | 0.978 | 0.955 |
| $20.0<\mathrm{TH} \leq 30.0$ | 0.968 | 0.982 | 0.955 |
| $30.0<\mathrm{TH} \leq 40.0$ | 0.970 | 0.985 | 0.956 |
| $40.0<\mathrm{TH} \leq 50.0$ | 0.971 | 0.987 | 0.956 |
| $\mathrm{TH}>50.0$ | 0.972 | 0.988 | 0.957 |

The stemwood and topwood BF values for RO related to DOB are very similar.
The following constants for bark factor could be used for simplicity with moderately approximate results, especially for a large number of sticks.

| Species | Stemwood | Topwood | Stemwood and Topwood |
| :---: | :---: | :---: | :---: |
| BO | 0.964 | 0.974 | 0.969 |
| RO | 0.976 | 0.986 | 0.980 |
| wo | 0.955 | 0.962 | 0.959 |

The prediction equations can be used to estimate BF at any DOB or TH. Since $\mathrm{BF}=\mathrm{DIB} / \mathrm{DOB}, \mathrm{DIB}$ can be estimated as $\hat{\mathrm{DIB}}=\hat{\mathrm{BF}} \bullet \mathrm{DOB}$ and DOB can be estimated as $\hat{D O B}=\mathrm{DIB} / \hat{\mathrm{BF}}$. Past DOB and DOB growth can be determined from past DIB growth as follows:

$$
\begin{aligned}
& \text { Past DOB Growth }=\text { Past DIB Growth } / \hat{\mathrm{BF}} \\
& \text { and } \\
& \text { Past } \mathrm{DOB}=\text { Present } \mathrm{DOB}-\text { Past DOB Growth }
\end{aligned}
$$

where past DIB growth might be obtained with an increment borer.
Specific uses of the prediction equations include: (1) estimation of the solid wood and bark volume of standing trees, (2) estimation of bark volume, or peeled volume from unpeeled volume, of felled tree sections, (3) growth studies, and (4) estimating tree form (e.g., Girard Form Class).

See Husch et al. (1982) for a detailed discussion on bark factors.

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